MODULATION TRANSFER FUNCTION, INFORMATION CAPACITY AND PERFORMANCE CRITERIA OF SCINTISCANS

Earle C. Gregg

Western Reserve University, Cleveland, Ohio

It was early recognized that all scintiscans possessed two important and fundamental parameters that might be used to evaluate the performance of any scintiscanning system: namely, the apparent contrast and apparent resolution. It was further recognized that the apparent contrast was controlled by the statistics of detecting and recording discrete events while the apparent resolution was a geometrical property of the system. Only recently has it become obvious that these parameters (and associated problems) are common to all systems producing visual images and that the knowledge and techniques developed in radiology, optics and communication engineering could be applied directly to scintiscanning.

The literature of scintiscanning (*1–3*) and radiology (*4*) is replete with examples of attempts to use these parameters individually or in some combination to produce a number or some function of numbers that could be used as a "figure of merit" or equivalent for the systems under consideration. These attempts are all characterized by differentially weighting the influence of either parameter in a rather arbitrary fashion depending upon the desires and objectives of the investigator making the proposal. Recently it was recognized that even the term "resolution" is rather arbitrary and subject to many different types of definitions and measurements, and the latest trend is to use the modulation transfer function (MTF) instead of resolution. Unfortunately, this trend has only added more confusion to the problem of performance criteria since a continuous function is involved instead of a finite number for "resolution."

In spite of this trend it is patently obvious that the subjective term "resolution"—most commonly defined as the reciprocal of the minimum detectable distance between two infinitely narrow lines—has been of some value for a quick appraisal of a system, and the term is still used in the literature. What has not been so obvious is that the determination of the resolution as defined above depends on both (a) the total number of events involved in the image (essentially the possible "contrast" in the system) since this determines the probability of seeing an edge or line and (b) the manner of presentation since the autocorrelation process in the viewing eye will partially add successive events and make it easier to detect a series of lines than a single line or a pair of lines with the same contrast. The MTF does not suffer from this complication since it is a geometrical property of the system and is theoretically measured for an infinite (nonstatistical) source. It also has many interesting and quite useful mathematical properties when compounding systems (*5*).

To develop the concept of performance criteria, we shall first accept the concept of resolution (r) as defined above recognizing that it is only an approximation and should be determined with large signal fluxes. Threshold contrast ($\Delta B/B$), on the other hand, is simply the minimum signal change (ΔB) required in a given area A to distinguish this area from equivalent areas in its surrounds (B). While this is obviously controlled by the statistics of the events per unit area making up B, the real question is the size of the area A which determines the total number of events involved and hence the probability that what is observed is real and not a statistical deviation. In the past, it has generally been assumed that the minimum observable area $A = 1/r^2$ and that, for the observance of one event only in a large field with a 99.7% chance that it is real, ΔB should be about 3 times the standard deviation in B. This latter number will vary somewhat depending on the size of the area and will become considerably smaller if the pattern is repeated rather than being a single spot. For our purpose we shall consider single spots only (non-correlated) since our ultimate concern will be the detection of one abnormality in a large field. The smaller values

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should of course be used when evaluating the influence of a series of closed contours on detectability.

The first attempt to provide a single number criterion for imaging systems (i.e. radiographs) was that of Hay (*7*) who proposed the product (cr) of the contrast sensitivity (the reciprocal of the threshold contrast) and the resolution which he called the information index. This seemed to be a rather fundamental approach since it can be shown that for purely statistical events and a fixed resolution (independent of the density of events), the index is directly proportional to the square root of the number of events detected (or recorded) per unit area.

That this formulation of a criterion is debatable on fundamental grounds becomes very obvious when one tries to apply it to the collimator of a scintiscanning system. Consider a straight-bore collimator of diameter D looking at a plane of activity some distance away. The resolution is obviously proportional to $1/D$ and the total detected signal to D^4. The contrast sensitivity is then proportional to $D^4/D^2 = D^2$, and the information index becomes proportional to D. This means that the index increases with the diameter leading to the absurd result that the best collimator is one with an infinitely large hole.

The next attempt was by the author (*4,8*) who, on the basis of information theory, proposed the use of information capacity which was of the form $r^2 \log (1 + c)$. This formulation obviously weights the influence of the resolution much more heavily than the contrast which in turn supports the subjective feeling that contrast is relatively unimportant when merely conveying information. Further, when c and r (still fixed) are evaluated in terms of the collimator diameter, curves of information capacity as a function of diameter are obtained (*8*) which show an obvious maximum for various collimators, and these maxima are those generally accepted as optimum by clinical experience. *It is only this type of subjective test that establishes the validity of any proposed criterion.* In addition, because of its formulation, the information capacity has other uses—particularly in determining the size and complexity of associated equipment such as image handling devices and computers.

The obvious weakness in the above formulations, however, is that they involve a fixed resolution. The proper solution depends on the use of the MTF, which is independent of signal, and this approach was used in a recent paper (*6*) which formulated the information capacity of a system in terms of the total received signal density and the over-all MTF of the system. It is the purpose of this paper to apply this formulation directly to scintiscanning and, as will be seen, this approach not only eliminates the use of resolution but also preserves the maxima mentioned earlier that are dictated by experience.

Information capacity as used in this treatment is the maximum rate at which information can be sent through a channel (system) having an average power limitation and disturbed by random noise. While most communication systems are concerned with time-rates, it is obvious for two-dimensional viewing that area may be substituted for time. In the case at hand, we assume the message to be the average density of events (counts per unit area) being viewed and the power limited channel or system with noise to be the actual detected stream of randomly varying particles from the source. For the capacity to be a maximum, we must assume the message to be "Gaussian" coded which merely determines the manner in which the source is distributed over the plane of view. A picture of such a source would only look like more noise. This interpretation of information capacity can be shown to be equivalent to the statistical mean information gained (or content) per unit area from one measurement of a signal plus noise both with an *a priori* Gaussian distribution. Thus we are not determining the information content of a totally viewed

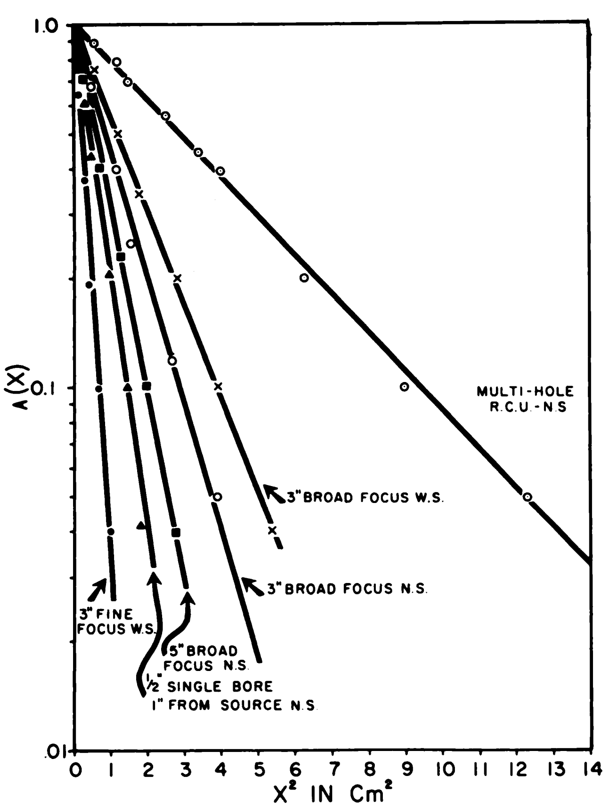

FIG. 1. Normalized line intensity A(x) as function of x^2 for various collimators. W.S. and N.S. represent "with scatter" and "no scatter" conditions. Multi-Hole R.C.U. line is data for scintillation camera with ratio circuit (2) and is graphed as $\tau(K)$ vs $100\ K^2$ to same numerical scales. Source was ^{131}I which was located at focal distances shown and 1 in. from multihole collimator.

field but rather the maximum amount of information (per unit area) that can be gained when using a particular system. Changes in the system or various systems can then be quantitated in these terms.

Now, the information capacity of any imaging system was shown (*6*) to be

$$I = \frac{1}{2} \int_0^{K_y^m} \int_0^{K_x^m} \log_2 N^2 \left(1 + H \frac{|\tau(K_y)\,\tau(K_x)|^2}{K_x K_y}\right) dK_x dK_y$$

where K_x and K_y are the spatial frequencies in the x and y directions, $\tau(K_x)$ and $\tau(K_y)$ are the MTF's in the x and y directions, H is $4/\pi^2$ times the signal-to-noise power ratio, $N^2 \approx 0.1$ is a constant determined by the statistical uncertainties desired, and K_x^m and K_y^m are determined by the condition

$$H \frac{|\tau(K_x^m)\,\tau(K_y^m)|^2}{K_x^m\,K_y^m} \geqq 10$$

For the isotropic case where $\tau(K_x) = \tau(K_y)$ [which is only approximately true for good mechanical rectilinear scanners with small steps in the y direction], the above expression was shown to reduce to

$$I = \frac{K_m^2}{2} \log_2 \frac{H}{10} + K_m \int_0^{K_m} \log_2 \frac{\overline{\tau(K)}^2}{K}\, dK$$

where K_m is determined by

$$H \frac{|\tau(K_m)|^4}{K_m^2} = 10.$$

This upper limit to the integral (K_m) is really that value of spatial frequency for a given signal-to-noise ratio beyond which any measurements (i.e. of "resolution") are fruitless because of statistics. This value of K_m will be called the "apparent" resolution since it is the reciprocal of the minimum resolvable distance between two bars under the signal-to-noise condition determined by H. Its actual numerical value will be smaller than that determined in practice by repeating bar patterns because of the assumption of only one pattern in the field. The variation with H, however, should be identical and has been verified by experiment (*6,9*).

While the application of these expressions to scintiscanning will be treated in detail later, it is important to note that the above equations apply only to the two-dimensional image for which the MTF is specified. In scintiscanning the MTF of the detector may be included provided the position of the object plane is noted since, in most systems, the MTF changes (sometimes violently) with distance from the collimator. It has been common practice to assume the object plane to be normal to and located at the focal "point" on the axis of a focusing collimator or at some distance fixed by clinical demands with an unfocused system such as a single bore, honeycomb or pinhole collimator. The signal-to-noise ratio should also be measured for a planar source located at this same point.

While it is possible to compute the final two-dimensional information capacity when viewing a three-dimensional volume with a collimator whose MTF is known for each plane or "slice" through the volume, neither the clinical sophistications nor uses at this time warrant the additional mathematical complications. However, in considering the use of such systems for viewing volumes with distributed activity, the information capacity of the final two-dimensional image will always be less than that specified for the plane with the "best" MTF, and the reduction will be greater for those detectors whose MTF's change more violently with depth in the volume.

MODULATION TRANSFER FUNCTION

There are many ways of specifying the geometrical image-forming characteristics of a system, but the most common are the point-spread function, line-spread function and modulation transfer function (*5*). While both the line spread and point spread are one-dimensional functions (intensity versus distance perpendicular to the line or radially from the point), the point-spread function alone implies that the field is isotropic and independent of direction (θ). Since the field may not be isotropic, it is generally preferred to specify the line-spread function along the chosen set of orthogonal axes (x and y or r and θ for a two dimensional field). However, in the isotropic case, the point-spread and line-spread functions are related mathematically since a line may be consid-

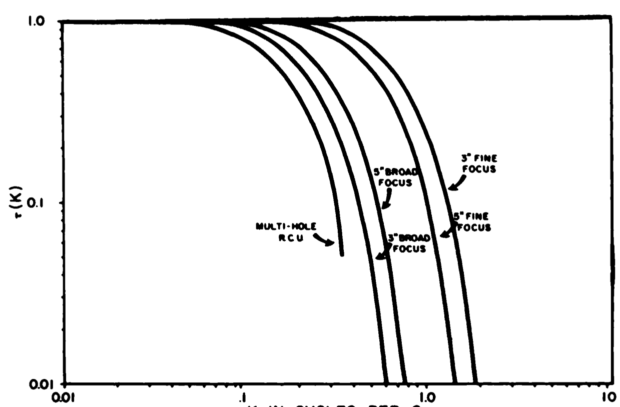

FIG. 2. Modulation transfer function $\tau(K)$ for various collimators. ^{131}I line source without scatter was located at focal distances shown and 1 in. from front surface of multihole collimator.

TABLE 1. WIDTH AT HALF MAXIMUM ("d") FOR VARIOUS COLLIMATORS*

Type	Condition	With scatter ^{131}I	With scatter ^{99m}Tc	Without scatter ^{131}I	Without scatter ^{99m}Tc
5-in. fine focus (265 holes)	Line at 5 in.	0.82 cm	0.74 cm	0.80 cm	0.61
	Point at 5 in.			0.76	
5-in. broad focus (85 holes)	Line at 5 in.	1.50	1.48	1.50	1.38
	Point at 5 in.			1.50	
3-in. fine focus (163 holes)	Line at 3 in.	0.86	0.81	0.72	0.70
	Point at 3 in.			0.70	
3-in. broad focus (31 holes)	Line at 3 in.	2.18	2.00	1.90	1.67
	Point at 3 in.			1.85	

* Discriminator window was between 315 and 415 kev for ^{131}I and between 105 and 175 kev for ^{99m}Tc.

ered to consist of a set of points. If $I(x)$ is the line-spread function for a line centered at $x = 0$ and $C(r)$ is the point spread function for a point at $r = 0$, then it may be shown (*10*) that $A(x) = 2\int_0^\infty C(r)\ (r^2 - x^2)^{-1/2}\ rdr$ and, inversely, that $C(r) = \frac{1}{\pi}\frac{d}{dr}\int_r^\infty \frac{A(x)\ rdx}{x(x^2 - r^2)^{1/2}}$. As mentioned, a more powerful and useful device than the line-spread function is the modulation transfer function $\tau(K)$ which, by definition, is the Fourier integral transform of the line-spread function:

$$\tau(K_x) = \int_{-\infty}^{\infty} A\ (x) \cos 2\pi Kxdx$$

where K is the spatial frequency coordinate (lines per unit distance). For an isotropic field it may be shown that $\tau(K) = 2\pi\int_0^\infty C(r)\ J_o\ (2\pi Kr)\ rdr$ and, inversely, that $C(r) = 2\pi\int_0^\infty \tau(K)\ J_o\ (2\pi Kr)\ KdK$ where J_o is the Bessel function of the first kind and zero order. The normalization conditions on the above equations are $\int_{-\infty}^{\infty} A(x)\ dx = 2\pi\int_0^\infty C(r)\ rdr = \tau(0) = 1$. A most interesting property of the MTF is that if $\tau_1(K_x)$, $\tau_2(K_x)$, $\tau_3(K_x)$ etc. are the transfer functions of each element in a series of operations on an image, then the final MTF becomes $\tau(K_x) = \tau_1(K_x)\ \tau_2(K_x)\ \tau_3(K_x) \ldots$ The same is so for $\tau(K_y)$.

Of immediate practical importance in scintiscanning is the fact that Gaussian functions will transform into Gaussian functions. If one has a Gaussian point-spread function of the form $C(r) = \frac{a^2}{\pi}e^{-a^2r^2}$ where $a^2 = \frac{2.78}{d^2}$ and d is the width at half-maximum, it can be shown by simple substitution in the above equation that $A(x) = \frac{a}{\pi^{1/2}}\ e^{-a^2x^2}$ which states that the line-spread function will be of the same form with the same width at half-maximum (see Appendix). Further, it may be shown that $\tau(K) = e^{-\pi^2K^2/a^2}$. Thus, if the line-spread (or point-spread) function is Gaussian, only the width at half maximum need be known since the analytic function is completely specified. It is to be noted that $\tau(K) = 0.0287$ for

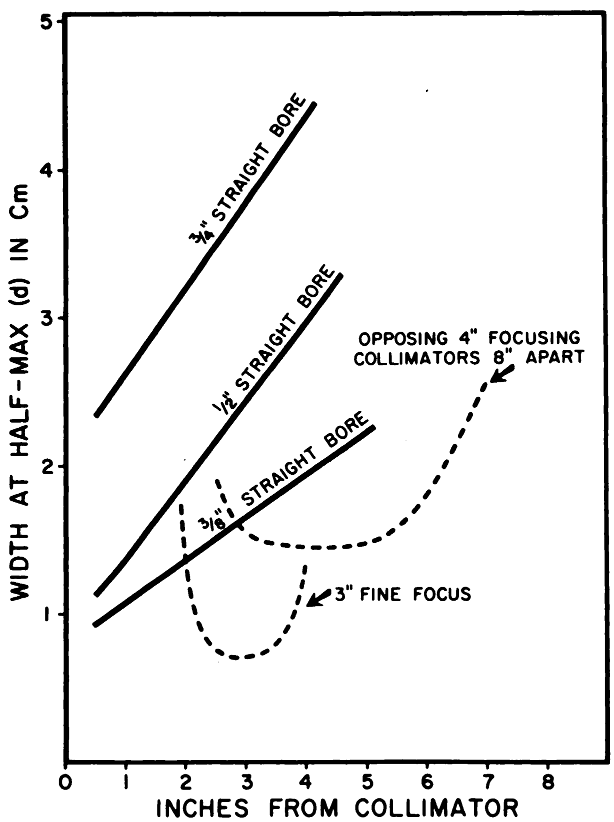

FIG. 3. Width at half maximum "d" for various collimators as function of distance from front face of collimators. ^{131}I line source was used without scatter.

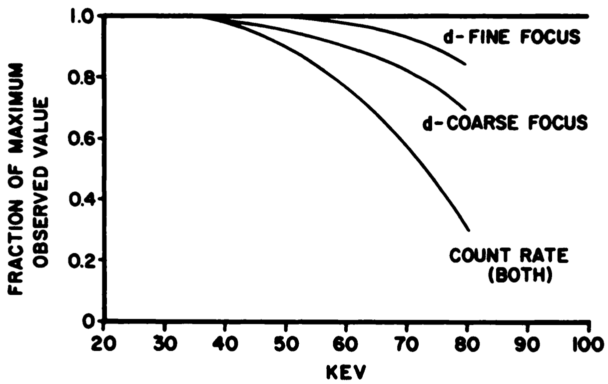

FIG. 4. Fraction of maximum observed "d" and integral counting rate for two different focusing collimators as function of low-energy discriminator setting. Line and planar sources of ^{197}Hg were used in scattering medium.

$K = 1/d$ which means that the (system) amplitude is down by a factor of about 35 for image spatial frequencies on the order of the resolution when the latter is defined by the reciprocal d value. From the above, it is obvious that if log A(x) is plotted against x^2, the function is Gaussian if a straight line results.

Figure 1 shows the results of determining the line-spread function of several collimators by scanning a 1/16-in. i.d. tube 12 in. long filled with radioactive material as the line source. It is obvious from the graphical results that the assumption of a Gaussian function is reasonably accurate for the collimators shown, and this was true for the majority of collimators and conditions investigated. In a few isolated cases, deviations from Gaussian were noted at large values of x, but, as will be seen, these only modify slightly the MTF at very small values of K and have little effect on the subsequent calculations of information capacity. Replotting published data on a scintillation camera (*2*) also shows a reasonable Gaussian function for this class of devices. This is demonstrated in Fig. 1 where for the Multi-Hole RCU the abscissae are units of 100 K^2 and the ordinates $\log_T(K)$ which produces the expected straight line for a Gaussian relationship. While Fig. 2 illustrates the MTF for a few of the collimators, the remaining data in Table 1 and Fig. 3 are presented in terms of the width at half maximum because of the close approximations to Gaussian functions. This approximation is also supported in Table 1 by the close agreement between the "d" values for line sources and point sources which should be identical for Gaussian functions. The influence of scatter was determined by inserting a 4-in. block of plastic between the line source and detectors. It is interesting to note that the effective "d" for two focusing collimators in opposition varies less with position of the source plane between the collimators than that for a focusing collimator alone, thus leading to a more uniform view of a volume.

Figure 4 shows the fraction of maximum integral counting rate observed when viewing a uniformly distributed source of ^{197}Hg with scatter for two different focusing collimators (both 3 in. diameter, 3 in. focal length as shown in Table 1) as a function of the lower discriminator cut-off energy. On an absolute basis, the coarse-focus counting rate and d values were 5 and 2.7 times larger than those for the fine-focus collimator. Other collimators produced very similar results which show that fair increases in counting rate can be obtained by opening the amplifier "window" without appreciably affecting the MTF.

MTF OF VISUAL PRESENTATIONS

As mentioned, for any system consisting of elements in series, the final MTF is the product of the MTF's of all the elements making up the system.

Now, the process of scanning is really one of converting a two-dimensional signal (i.e., a signal depending on x and y) into a one-dimensional signal varying with time in such a manner that at any given time both x and y positions are known. Generally, for rectilinear mechanical scanners and television scanning, the scan is continuous in x and stepwise in y. This means that if the electronic circuitry has an appreciable time constant, the MTF along both axes will become worse with the x-axis showing the greatest effect. By appreciable is meant a circuit time constant on the order of or larger than the time required to move the scanner about one "apparent resolution" distance as specified previously. "Scalloping" of the edge of a pattern occurs when a rate-meter (or any time-averaging) circuit with an appreciable time constant is used and adjacent line scans are made in opposite directions. A more accurate measure of this effect may be obtained by determining the frequency response of the electronic circuitry (amplitude vs. frequency) since it can be shown rather simply that the equivalent MTF (in the x-direction) is this frequency response replotted with the frequency axis converted to spatial frequency (K) by dividing by the scan velocity in the x-direction. The same considerations hold true in the y-direction provided the steps in y are much smaller than the apparent resolution and a continuous curve approximation is made to the step function involved. It is to be noted that the electronic frequency response referred to is not that bandwidth required to handle pulses but rather that required to handle or compute the time average of a random sequence of pulses.

Even if the circuit has no time constant, the size of the recording spot must be taken into account since it too has an MTF (*3*). This may be obtained by simply determining the point or line-spread function of the spot intensity (or "dot" distribution) and transforming this into the MTF by the mathematical techniques discussed previously. To minimize the line structure due to the steps in the y-direction and/or to approximate the spatial distribution of the probability of detection by the collimator, it has been common practice to "data-blend" in photoscanning by using a recording spot approximately as large as the field of the viewing collimator. Whether one uses a "Gaussian" spot of weaker intensity for density addition through super-position or a "Gaussian" spread of small dark spots for transmission subtraction is of little consequence. The net result is a high frequency degenerated MTF in both x- and y-directions that is the square of the MTF of the collimator alone.

Further, for producing a uniform record when viewing a uniform field, the concept of a "Gaussian" spot of any size is incorrect since it has been shown (*11*) that the ideal shape for such a spot is a cosine-squared dependence with half-overlap on the basis that $\cos^2\theta + \sin^2\theta = 1$. Practically, however, the Gaussian-shaped spot is probably a reasonable approximation to a cosine-squared distribution.

Another reason for a "bell-shaped" spot that has been recently advanced (*14*) is that the eye may be perturbed by a small sharp spot that has higher spatial frequency components than the collimator itself. While minification to the extent that the MTF of the eye would approximately match that of the collimator on the viewed record would eliminate such an effect, it was proposed that one could possibly circumvent minification by using a spot shaped so that the higher frequency components would be absent. Ideally then one would like a spot shaped so that $\tau(K) = 1$ for all frequencies up to some arbitrary cut-off, say K_1, determined by the collimator. This would in essence be a low-pass filter commonly used in communication engineering to eliminate unwanted noise components. From the previous relationships, it is simple to show that the shape of such a spot is

$$C(r) = 2\pi\int_0^{K_1} J_o\ (2\pi Kr)\ KdK = \frac{K_1}{r}\ J_1\ (2\pi K_1 r)$$

where J_1 is a Bessel function of the first kind and first order. A graph of this is a "bell-shaped" curve with alternating and successively diminishing maxima and with the first zero at $r = 3.83/2\pi K_1$. It is obvious from the phase requirements of the ideal spot shape (which imply storage and spatial addition of positive and negative components of all recorded spots) that any practical spot shaping used to date is a very rough approximation that compromises between pleasing the eye and degenerating the over-all MTF. "Out-of-focus" and diffusion techniques produce similar effects.

Theoretically, it is possible to go further and propose a two-dimensional filter that will actually correct for the over-all MTF and restore the original unmodified isotope distribution. For very large numbers of detected events, if $\tau(K)$ is the system MTF, then the filter should have an MTF equal to $[\tau(K)]^{-1}$ since the product would then be unity. However, noise and statistically fluctuating signals render this approach most difficult both theoretically and experimentally, and such complications will not be discussed here.

We have preferred the approach of preserving the original distribution as seen by the collimator by using a small sharp dot with a system that has a spatial frequency bandwidth much greater than that of the collimator. We then later adjust the over-all characteristics on replay by optical and electronic techniques. For preservation of the collimator MTF with a mechanical scanning system, the best shaped "spot" (or distribution of spots) is one that is very narrow in the x-direction (practically a "line") with a cosine-squared distribution in the y-direction of such an amplitude that half-overlap occurs between adjacent y scan lines. Needless to add, the y steps should be small ($<$ ⅓) compared to the apparent y resolution. An obvious disadvantage to this system is that at low counting rates each count would then record as a small line. To circumvent this annoyance, the latest system developed in these laboratories produces an image the same size as the object and uses a single, intense and very small recording spot (about 0.005 in. and now termed a microdot) per detected count with a cosine-squared y-axis displacement modulation such that half-overlap occurs between adjacent scan lines. The time duration of the intensified spot is very short compared with the modulation period so that an almost circular spot is produced for each count. Since the actual size of the recording spot relative to the image size is really dictated by the degree of spot coincidence (in space) that can be tolerated, very small spots are required to accommodate high field densities. However, it has been found that the eye prefers somewhat larger spots at low field densities because a few very small spots do not produce an appreciable stimulus, and for this reason the spot size has been made adjustable over a small range. Generally, a spot size chosen for an acceptable picture by the eye is still well within the restrictions of the final MTF and probability of spatial coincidence. The over-all system produces a truly random-appearing field with the final x and y MTF's very close to that of the original collimator and with a recorded spot density (spots per unit area) equal to the original count density.

As with all methods of recording, the more serious problem is not the relationship between the recorded spot density (in terms of counts per unit area) and the original detected count density but rather the linearity and range of the final image as seen by the eye, by a densitometer or by a TV scanner relative to the original image. This arises primarily because of nonlinear photographic density addition of spots when used in a superposed system or the inability to produce a truly saturated spot when using the "microdot" technique.

Figure 5 is a graph of the measured photographic density as a function of count density for our microdot technique while Fig. 6 is the record obtained when scanning a line source in both the x- and y-directions. Measurements with a long rectangular slit densitometer produced a Gaussian line-spread function with a width at half maximum of 0.65 cm for both directions of travel which is about 5% larger than that of the collimator alone at the given source-to-collimator distance. The line spacing in the y-direction was 0.2 cm.

Figure 7 is a composite of four scans made of the same phantom with approximately the same total recorded counts but with different "d" values (for

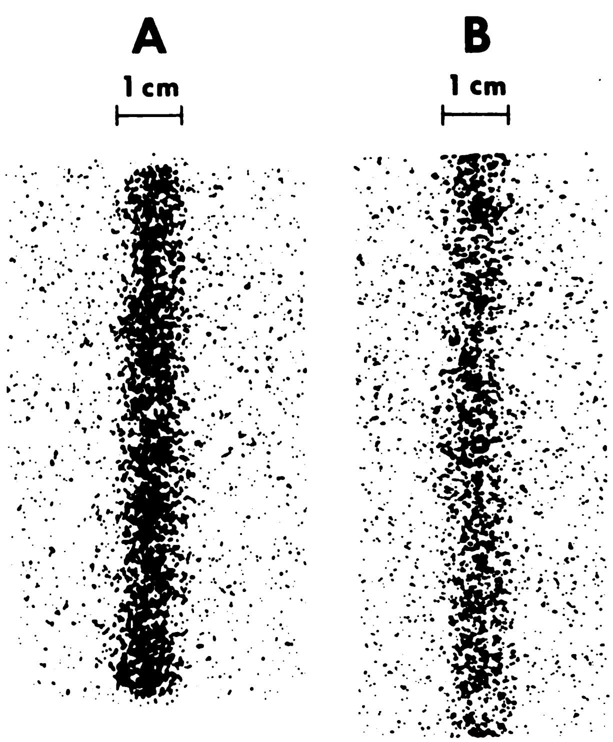

FIG. 6. Reproduction of final microdot film record of line source perpendicular (A) and parallel (B) to x scanning direction.

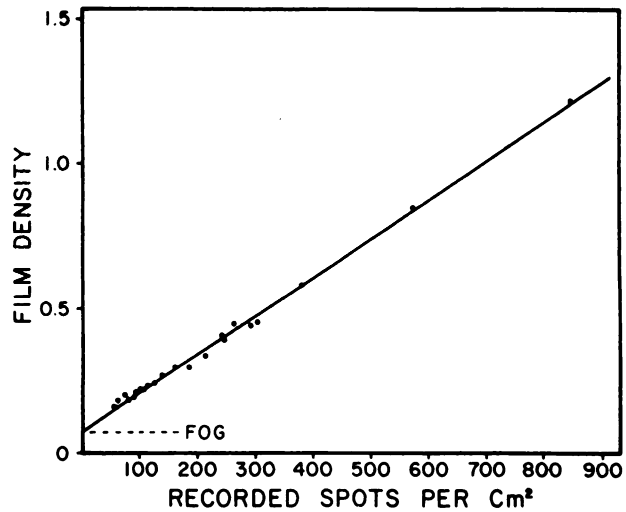

FIG. 5. Photographic density vs. recorded counts/cm² for microdot technique.

^{99m}Tc) as shown. Actually, the total counts recorded for d = 3 cm are about 30% higher than those for d = 0.7 cm, with the remaining two proportionately between. The phantom consisted of a plastic box containing two hollow triangular prisms of equal viewed size (short base 3 in.) but with one three times the depth of the other. In addition, each prism had a nonradioactive void (plug) 0.5 in. in diameter located in the center, and both prisms were filled in series through a 1/16-in.-diameter tube which can be seen in the scan with the smallest "d" value. The change of discernible detail with "d" value is quite apparent in these scans.

INFORMATION CAPACITY

While the more exact expression for information capacity given previously may be evaluated for each and every case, several simplifying assumptions may be made that are reasonably close for most scintiscanning systems or cameras:

1. The recorded image or data is isotropic.
2. The line spread function of the final image is Gaussian of width "d" at half maximum.
3. The statistics are determined by the source only, and there is negligible extraneous noise, leakage and scatter.
4. There is a linear relation between the original (desired) density of events and final (observed) density or output parameter.

Assuming the expression $\tau(K) = e^{-\alpha^2 K^2}$ where $a^2 = \pi^2 d^2/2.78$ and substituting in the previously given expression, we find

$$I = \frac{1.44\,K_m^2}{2} \ln\left(\frac{H}{10}\right)$$

$$+\ 1.44\,K_m \int_0^{K_m} \ln\left(\frac{e^{-2\alpha^2 K^2}}{\overline{K}}\right) dK \text{ bits/unit area.}$$

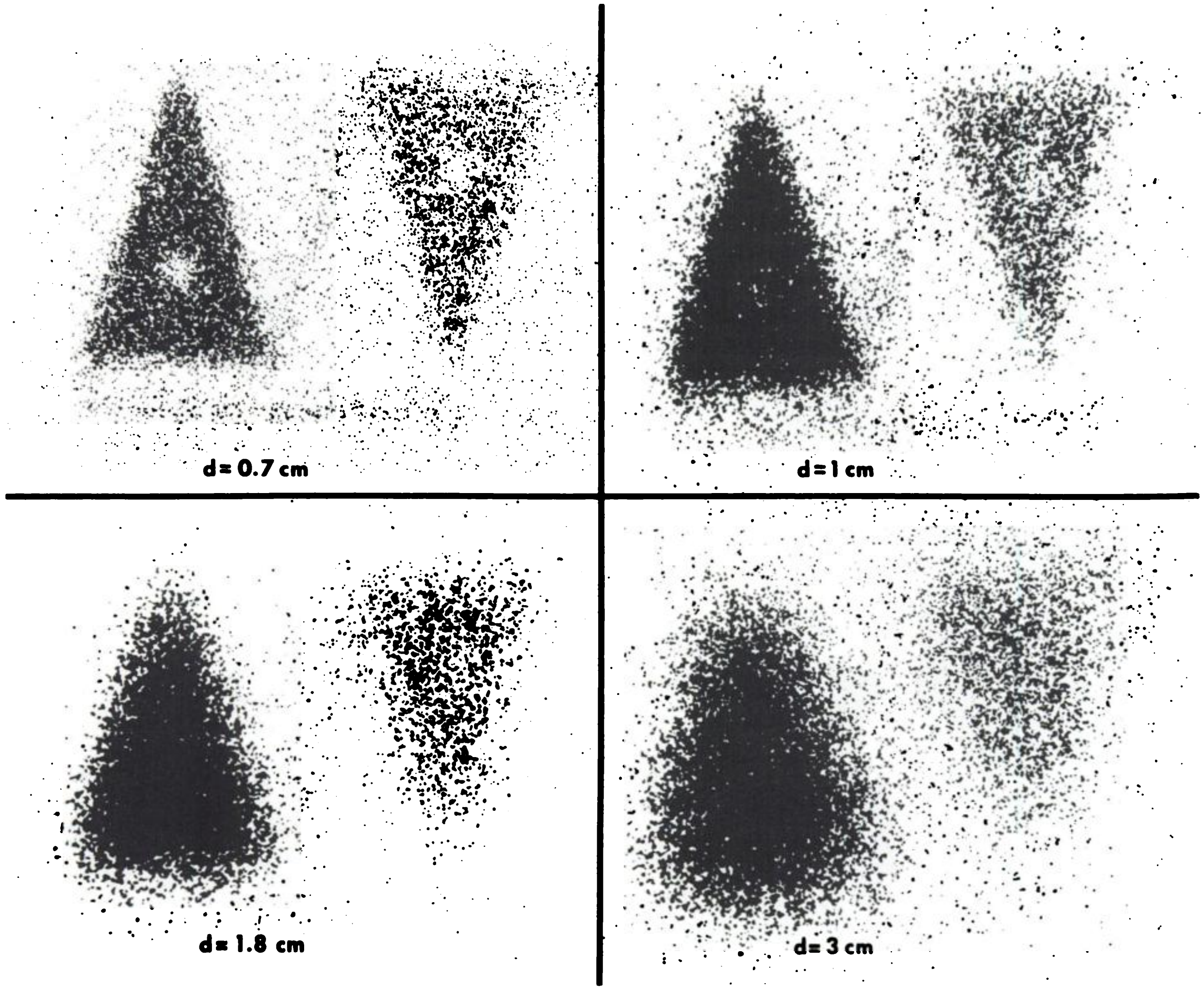

FIG. 7. Composite of four scans of same phantom with approximately same number of total counts but different "d" values as shown.

where $$\frac{H}{10} = \frac{K_m^2}{e^{-4\alpha^2 K_m^2}} = \frac{4}{\pi^2} R_o$$

and R_o is the signal-to-noise power ratio per unit area. Letting S_o be the detected (and hence recorded) counts per unit area, we see that the rms noise in the signal is $(S_o)^{1/2}$ and the signal-to-noise power ratio $R_o = S_o$.

Evaluating and reducing the above expression (see Appendix), we find

$$I = 1.44\, K_m^2\, [1 + 4.7\, d^2 K_m^2]$$

and

$$\log_{10} S_o = 2 \log_{10} K_m + 6.15\, d^2 K_m^2 + 2.4.$$

For spatial frequencies in lines per centimeter, d in centimeters and S_o in counts per square centimeter, I will have the units of bits per square centimeter. It is important to note that all measurements of distance or area are referred to the original object plane, and any geometrical scaling of the final image must be taken into account.

Numerically evaluating the above two expressions for various values of d and K_m, it is ultimately possible to obtain the generalized graphs shown in Figs. 8 and 9 which show both I and K_m as a function of S_o for various values of d. As might be expected, both log I and K_m increase with log S_o with the greater increases occurring for the smaller d values.

While it is possible to evaluate S_o in terms of the distributed radioactivity and the geometrical efficiency of the collimator, the graphical formulations as given are much more convenient since they apply directly to the observed counting rate, time (or velocities) of scanning and the over-all MTF for the given distance of the source from the collimator. Likewise, should the recorded signal contain extraneous noise, the same curves may be used with the simple substitution of the observed signal-to-noise power ratio per unit area R_o for S_o.

As an example of the noiseless case, consider a mechanical scintiscanning system that has an aver-

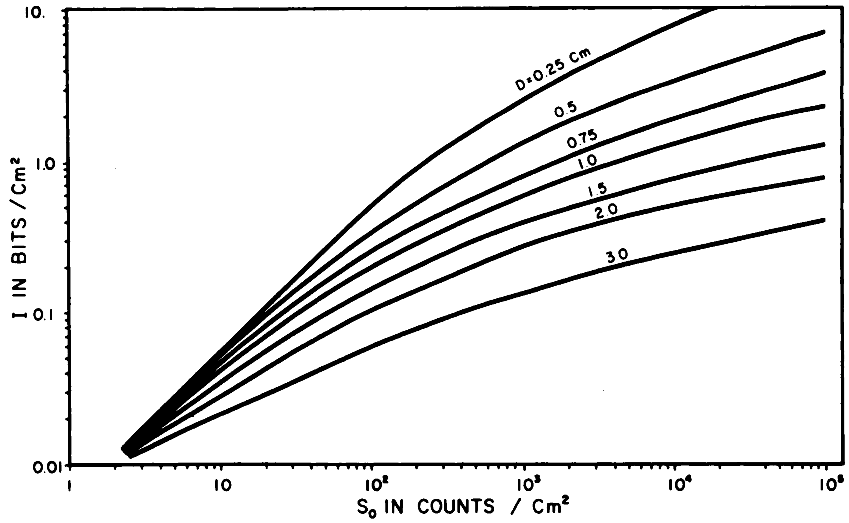

FIG. 8. Information capacity as function of recorded (detected) count density for various "d" values.

age counting rate of 1,000 cpm for a given uniform field of activity, an x scan speed of 20 cm/min with y steps of 0.2 cm and a final image of "d" value of 1 cm. S_0 then becomes 250 counts/cm^2 which will produce $K_m = 0.37$ lines/cm and $I = 0.33$ bits/cm^2. If S_0 should be increased to 500 counts/cm^2 by increasing the specific activity of the source or halving the scan velocity, the information capacity would become 0.48 bits/cm^2 and $K_m = 0.39$ lines/cm.

As another example, consider a typical scintillation camera with a honeycomb collimator looking at a distributed planar source such that the effective "d" value is 0.85 cm and the counting rate is 100,000 cpm for a field of 400 cm^2. This latter figure produces a density of 250 counts/cm^2/min. For a total scan time of 10 min, we then obtain an information capacity of 0.95 bits/cm^2 and a value for K_m of 0.33 lines/cm. An increase of a factor of four in intensity or scan time would produce $I = 1.4$ bits/cm^2 and $K_m = 0.6$ lines/cm, while a decrease of a factor of four would produce $I = 0.5$ bits/cm^2 and $K_m = 0.4$ lines/cm.

It is fairly obvious that a scintillation camera with a honeycomb collimator may be considered to be a parallel aggregate of single-bore detectors, all operating simultaneously, with the net result of shortening total-scan (or observation) time for a given field and that all multidetector scanners (e.g. Dynapix, Autofluoroscope, etc.) may be treated in somewhat the same manner. However, it is most important to note that the information capacity in all cases is logarithmically—not linearly—related to the final recorded count density and this, in turn, points out some fundamental differences between accumulating information in series by a mechanical scanner and in parallel by a camera.

Now, the total information possible from any given record is IA_s where A_s is the field of view. For a mechanical scanner with a fixed counting-rate (R), a scanning velocity (v_x) and a line step distance (d_y), the information capacity I is determined by the logarithmic dependence on $S_0 = R/d_y v_x$. However, the total possible information is linearly proportional to the total time of scan (T_s) since this determines A_s. This arises because the information obtained from different fields of view is different and linearly additive. On the other hand, for a scintillation camera with a fixed field of view, the total scan time determines the total count N in the complete field of view which determines only $S_0 = N/A_s = RT_s/A_s$ where R is the counting rate for the whole detected field. Thus for a camera the total possible information is only logarithmically dependent on the total scan time which occurs because of the redundancy when looking at the same areas. The information obtained this way is not linearly additive since, in essence, longer times merely mean more counts per unit area which change only the signal-to-noise ratio in the logarithmic term.

The fact that the formulation for capacity used

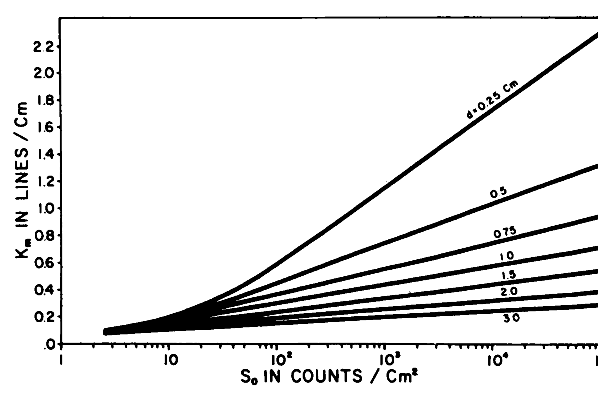

FIG. 9. Apparent resolution K_m as function of recorded (detected) count density for various "d" values.

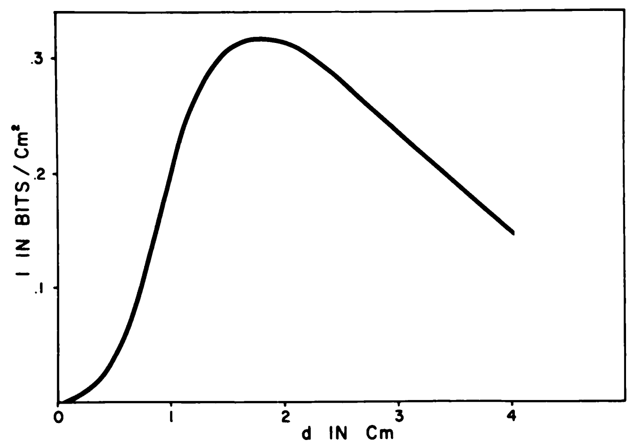

FIG. 10. Information capacity as function of "d" for collimator viewing same uniform planar source.

above also leads to an optimum collimator bore size may be seen graphically. Consider a typical single-bore collimator (note that most other style collimators may be compounded from such a geometry) viewing a field of activity such that it has a "d" value of 1 cm and, for the time and activity involved, records 100 counts/cm². This produces an information capacity of 0.2 bits/cm². If, now, the bore diameter were halved, "d" would become 0.5 cm and the new S_o would reduce to 6 counts/cm² since the detecting efficiency varies as the fourth power of the bore diameter. This produces $I = 0.031$ bits/cm². If the bore size were doubled, the new "d" would become 2 cm, S_o would be 1,600 counts/cm², and I would become 0.31 bits/cm². Repeating this process for various "d" values produces the curve shown in Fig. 10 which demonstrates an obvious maximum of about 0.32 bits/cm² at $d = 1.8$ cm. While the assumption that the "d" value is linearly related to the bore diameter is reasonably correct over a fair range of diameters according to the data on Fig. 3, deviations from linearity would only affect the "d" scale and not the general conclusions. For a 4-in.-thick collimator looking at a source plane 4 in. away from its front face, the above "d" value corresponds to a bore size of 0.9 cm or 0.39 in.² which agrees reasonably closely with the optimum size predicted by the more naive formulation proposed originally (*8*). While the check between the two methods of approach shows that the calculation of information capacity is not too sensitive to the shape of the MTF (other than some cut-off value of K related to the "resolution"), the more sophisticated approach used here leads to simpler generalized curves as well as a graphical relationship between apparent resolution and signal density.

FIGURE OF MERIT

Although this author deprecates the usefulness and significance of a single figure of merit for comparison of complex systems, there have been numerous attempts at such reported in the literature, primarily by those interested in instrumentation and in comparing their own approach to the over-all problem.

Besides its obvious usefulness in quantifying system or component performances, the information capacity may also be used as such a "figure of merit" for various systems provided all are measured under identical conditions. While it is rather easy to specify that the information capacity of the final record be obtained for scanners viewing a planar source of a given activity (say, 0.2 $\mu c/cm^2$) and given isotope (say, ^{131}I or ^{99m}Tc) in the same plane for which the MTF (or "d" value) was determined, not so easily specified or compared are two additional parameters of clinical importance: namely the maximum possible area of the scan and the time required to produce a scan possessing sufficient detail. The time from the beginning to the end of the observation period is of importance not only because of patient comfort but also because of the effect of patient motion on decreasing the high-frequency components of the MTF. This latter point has been well known in diagnostic radiology (*12*) and recently reported for scintiscanning (*13*). One must remember, however, that the effect of motion will be different depending on the type of scan involved. In the case of the camera, the MTF over the whole field will be affected because of continuous signal integration, while for a mechanical scanner it will be affected only at the position on the record when the motion occurred.

On the other hand, the maximum possible area of scan is complicated by the fact that it is fixed for cameras and unlimited for mechanical scanners. Since the importance of the time and possible area of scan is a clinical value judgment and hence not subject to insertion in any reasonable analytic formula without creating undue and arbitrary weighting, it is suggested for those who wish a single figure of merit to use the information capacity as determined above when scanning a field of fixed size (say, 10 cm $\times$ 10 cm) for a fixed period of time (say, 15 min). This could also be listed as a function of photon energy to show the range of the instrument provided the MTF's are also measured for the various energies.

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APPENDIX

1. Line-Spread Function and MTF of a Gaussian Point-Spread Function.

Given

$$C(r) = C_o e^{-2.78r^2/d^2} = C_o e^{-a^2r^2}$$

Normalizing

$$2\pi C_o \int_0^\infty e^{-a^2r^2}\, r dr = 1$$

$$2\pi C_o \left[\frac{-1}{2a^2}\right]\left[e^{-a^2r^2}\right]_0^\infty = 1$$

Thus

$$C_o = a^2/\pi$$

By definition

$$A(x) = 2\int_x^\infty \frac{a^2}{\pi} e^{-a^2r^2} (r^2 - x^2)^{-1/2}\, r dr$$

Substituting

$$y^2 = r^2 - x^2$$

Then

$$A(x) = \frac{2a^2}{\pi} e^{-a^2x^2} \int_0^\infty e^{-a^2y^2}\, dy$$

Thus

$$A(x) = \frac{a}{\pi^{1/2}} e^{-a^2x^2}$$

Check

$$\int_{-\infty}^{\infty} A(x)\, dx = \frac{2a}{\pi}\int_0^\infty e^{-a^2x^2} dx = \frac{2a}{\pi^{1/2}}\; \frac{\pi^{1/2}}{2a} = 1$$

By definition

$$\tau(K) = \int_{-\infty}^{\infty} \frac{a}{\pi^{1/2}} e^{-a^2x^2} \cos 2\pi K x\, dx$$

$$\tau(K) = \frac{2a}{\pi^{1/2}}\left[\frac{\pi^{1/2} e^{-\pi^2K^2/a^2}}{2a}\right] = e^{-\pi^2K^2/a^2}$$

Note that

$$\tau(K) = 1 \text{ for } K = 0.$$

2. Reduction of Information Capacity Formulation.

Given

$$I = \frac{K_m^2}{2}\log_2\left(\frac{H}{10}\right) + K_m \int_0^{K_m} \log_2 \frac{\overline{\tau(K)^2}}{K}\, dK$$

$$H\,\frac{|\tau(K_m)|^4}{K_m^2} = 10 = \frac{4}{\pi^2} S_o$$

and $\tau(K) = e^{-\alpha^2K^2}$ where $\alpha^2 = \pi^2 d^2/2.78$

Substituting and converting to natural logarithms

$$I = \frac{1.44\, K_m^2}{2} \ln\left(\frac{H}{10}\right) + 1.44\, K_m \int_0^{K_m} \ln\left(\frac{e^{-2\alpha^2K^2}}{K}\right) dK$$

$$= \frac{1.44\, K_m^2}{2} \ln\left(\frac{H}{10}\right) + 1.44\, K_m \int_0^{K_m} (-2\alpha^2K^2 - \ln K)\, dK$$

$$= \frac{1.44\, K_m^2}{2} \ln\left(\frac{H}{10}\right) + 1.44\, K_m\left[-\frac{2}{3}\alpha^2K_m^3 - K_m \ln K_m + K_m\right]$$

$$= 1.44\, K_m^2\left[\frac{1}{2}\ln\left(\frac{H}{10}\right) + 1 - \frac{2}{3}\alpha^2K_m^2 - \ln K_m\right]$$

Now

$$\frac{H}{10} = \frac{K_m^2}{e^{-4\alpha^2K_m^2}}$$

or

$$\ln (H/10) = 2 \ln K_m + 4\alpha^2K_m^2$$

Thus

$$I = 1.44\, K_m^2\left[\ln K_m + 2\alpha^2K_m^2 + 1 - \frac{2}{3}\alpha^2K_m^2 - \ln K_m\right]$$

$$= 1.44\, K_m^2\left[1 + \frac{4}{3}\alpha^2K_m^2\right]$$

$$= 1.44\, K_m^2\left[1 + 4.7\, d^2K_m^2\right]$$

Likewise, substituting $\frac{4S_o}{\pi^2}$ for $\frac{H}{10}$ in the above expression, we find

$$\log\left(\frac{H}{10}\right) = \ln S_o + \ln\left(\frac{4}{\pi^2}\right) = 2\ln K_m + 4\alpha^2K_m^2$$

or

$$\ln S_o = 2 \ln K_m + 4\alpha^2K_m^2 - \ln\left(\frac{4}{\pi^2}\right)$$

Substituting for α and converting to base 10, we find

$$\log_{10} S_o = 2 \log_{10} K_m + 6.15\, d^2K_m^2 + 2.4$$

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The Society of Nuclear Medicine has set aside Thursday afternoon, June 27, 1968, from 1:30 to 5:00 pm for a nuclear medical technologists program at the 15th Annual Meeting in St. Louis, June 27–30, 1968.

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