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Information Capacity of Scintiscans[1]

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This paper is an attempt to apply present day concepts of information content and information capacity to scintiscanning in order to compare various techniques and possibly to optimize any one technique. It must be recognized that such a development can be very complicated and approximations are necessary for a practical solution. Such approximations will be noted and their influence discussed at the time of application. Further, no attempt will be made to assess philosophically the significance of "bits" of information or systems having an information capacity of, say, 100 "bits per square inch" (or "bits per second") —this being more a matter of practical judgement through experience with a number of systems with various capacities.

From the basic tenets of information theory, it is obvious that any precise message like the letter "S" has an infinite information content. If we now look at "noise" as a message with a random distribution of probabilities, it follows that any one sampling will not produce the desired information (*i.e.* the magnitude and shape of the distribution) but rather will only have a certain probability that it is the desired information. It has been shown (1 - 10) that the information content of a gaussian distribution is measured by the logarithm (to the base 2) of its variance. When the base 2 is used, the content is measured in "bits" (or "binits") where one bit is the amount of information gained by knowing with certainty which of two alternate choices will occur. Thus, if we have a random distribution of black dots on a white plane with an average density of S per unit area and a variance of $\sqrt{SA}$ in a typical sampling of area A, the information content C_g of such a single sample is

$$C_g = \log_2 \sqrt{SA} \text{ bits.}$$

Now, each separate sampling adds more information, and since information from independent measurements of the same signal are additive, and there are A^{-1} sample areas per unit area, then we may say that the information content

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per unit area is

$$C_g = A^{-1} \log_2 \sqrt{SA} \text{ bits/unit area} \tag{I}$$

when the signal is "scanned" with an aperture of area A. Since A^{-1} is a more rapidly varying function than the logarithm, it follows that even though the information content for one sample area diminishes as the sampling area gets smaller, the information content per unit area increases. Thus, a large number of determinations of a unit area with a very small test area will tell us more about the distribution than one measurement of the unit area itself. This effect might be considered a resolution weighting effect, since making the test area smaller will not reveal more about the average value, but rather will produce finer detail about the shape of the distribution itself. Information content obviously involves both of these parameters. It is important to note that negative logarithms are not allowed, since they imply a choice of less than one "bit" which is meaningless. Fractional bits per unit area are, however, permissible.

A second term of importance is "information capacity" which is a measure of the amount of information that a given system may handle. Consider the case where we have a similar black-dot presentation consisting of a signal of average value S per unit area (*i.e.* density) containing "noise" with an rms variation of N in each scanning area A. Information theory (*2, 3, 4, 5, 6, 8*) then states that the information capacity of the system when decoding a distribution by scanning with area A is

$$Cc = A^{-1} \log_2 \left[1 + \frac{KS}{N}\right] \cong A^{-1} \log_2 \left(\frac{KS}{N}\right) \text{ bits/unit area} \tag{II}$$

for $\frac{KS}{N} >> 1$.

This formula holds for a mean square limited system which has no intersymbol interference. It thus applies only when the process of scanning is such that the signals in adjacent areas do not contribute to the signal from the area being scanned. However, since the log term is slowly varying, moderate interference will change the results only slightly. K is a constant that essentially determines the risk of error involved (10, pg. 148). For $K = 1/3$, one would expect about three sampling areas at random in 1000 such areas to have the minimum detectable signal; for $K = 1/5$, the chance would be about 1 in $10.^6$ While we shall use $K = 1/4$ as a reasonable criterion, the fact that $K = 1/4$ for the eye (12) is fortuitous and has nothing to do with the problem at hand. It is interesting that if we consider the signal to be the average value of a gaussian source as before and the noise to be the variance in the signal itself, then

$$N = \sqrt{S/A} \text{ and } Cg = Cc \text{ for } K = 1.$$

Thus, for these circumstances, the information capacity per unit area is equal to the information content per unit area. It is important to note that the above equation for N results from the fact that the rms deviations in the average number per unit area S becomes larger as the test area becomes smaller. This is a well known effect in the scanning of optical images (Selwyn Coefficient, see 6). Typical computed values of information capacity (6, 7) are 10^6 bits/cm² for film,

1.9×10^5 bits/cm^2 for the eye at 25 cm viewing distance and brightness of 100 millilamberts and 260 bits/cm^2 for one frame of a 20 inch television picture, or 0.6×10^6 bits/frame for any television picture.

If we could convert the signal seen by a scanning aperture to a signal varying in time (*i.e.* an electrical signal along a cable), it can be shown that the bandwidth required is

$$W = \frac{1}{A} (dA/dt).$$

This equation implies that resolution and bandwidth are related and that if a given recorded scintiscan (*i.e.* a "photoscan") is replayed by television techniques in order to enhance contrast, etc., the bandwidth of the electronic system could limit the resolution if it were too small. Actually, most black and white television systems have a horizontal bandwidth of 4 megacycles, while that required for a typical single-bore scintiscan is on the order of 100 kilocycles. One must not confuse this bandwidth with that required to accommodate the single pulses that may be used to construct a picture.

Let us now consider the case where we have a gaussian signal with an average S per unit area and added gaussian noise of average N per unit area and ask for the information capacity of a system that scans with area A. The observed variance in A would be

$$\sqrt{\frac{S+N}{A}}\ .$$

$$\text{Thus, } Cc = A^{-1} \log_2 \left[1 + \left(\frac{K^2\, S^2\, A}{S+N}\right)^{\frac{1}{2}}\right] \text{ bits/unit area} \qquad \text{(III)}$$

In this case, the information capacity will always be less than the information content of the message unless N = O, as might be expected. Equation III is applicable to scintiscanning where the source consists of random signals from radioactivity and the noise consists of random signals from scatter, leakage and background. The function of the collimator interposed between the source and detector is to identify and locate the detected signal, but in doing so it reduces considerably the amount of possible detected signal. All radioactive sources radiate into a 4π solid angle, but any collimator will detect only a very small fraction of this due to its small solid angle of acceptance. Thus, purely geometric considerations demand that the information capacity of the detecting system be far less than the information content of the distributed radioactive source. Most collimators are simply a hole in a thick block of lead (straight bore collimator) or some arrangement of holes (focussing and honeycomb collimators). Tapered holes may be closely approximated by straight bores and pinholes will be shown later to follow similar considerations and formulations.

Consider now a source of radiation distributed in a plane and a collimator that has an effective detecting area A_p on the plane of activity. If S^* is the detected source signal (*i.e.* number of counts in a given time due to the viewing aperture) and N^* the disturbing noise signal received in the same time, then the average detected signal and noise per unit area are S^*/A_p and N^*/A_p. Substitution of these in equation III leads to

$$Cc = (A_p)^{-1} \log_2 \left[1 + \left(\frac{K^2 (S^*)^2}{S^* + N^*} \right)^{\frac{1}{2}} \right] \tag{IV}$$

This formulation allows a convenient separation of projected aperture size and sensitivity. Now for a single bore collimator of true bore area A viewing a plane of specific activity σ disintegrations per unit area per unit time, for a time τ, it will be shown that $S^* = K_1 \sigma A^2 \tau$, where K_1 is a geometrical constant. Likewise, $N^* = (K_2 \sigma A^2 + K_3 \sigma + R_b) \tau$ where the separate factors are scatter, leakage and background. K_2 and K_3 obviously involve integration over the whole area of activity, since the total activity in the plane contributes to both. Further, scatter involves the volume viewed by the collimator (above and below, if the plane is immersed in a scattering medium) since the scattering points must be seen by the collimator. The leakage coefficient K_3 involves only the geometry and attenuation of the shield. Further, if we scan a large area A_s in time T_s then

$$\tau = T_s A_p / A_s = T^1 A_p = K_4 A T^1 \quad \text{where } A_p = K_4 A$$

and T^1 is the time per unit area of the scan. This formulation assumes that the whole area A_s is scanned but is not concerned with the details (*i.e.* the amount of overlap, etc.). Substitution of the above relations in IV leads to the information capacity C_{sb} of a single bore collimator as

$$C_{sb} = \frac{1}{K_4 A} \log_2 \left[1 + \left(\frac{C_1 A^5}{A^2 + C_2} \right)^{\frac{1}{2}} \right] \text{bits/unit area} \tag{V}$$

where $C_1 = \dfrac{K^2 K_4 K_1^2 \sigma T^1}{K_1 + K_2}$ and $C_2 = (K_3 \sigma + R_b)/(K_1 + K_2) \sigma$.

This equation was programmed on an IBM 1620 computor and some of the results are shown in Figs. 1-3, for $K_4 = 4$, $K = \frac{1}{4}$, and various representative values of C_1 and C_2. It is obvious that the information capacity passes through a maximum as A is changed and that the optimum value of A and the corresponding information capacity depend upon the value of the constants chosen. For example, if an optimum collimator is being used and the time of scan reduced, the only way to return to very nearly the same capacity is to increase the source strength. On the other hand, if the source strength is not changed, then the aperture must be increased in order to maximize the capacity for the reduced scan time. However, the same (larger) value of capacity will never be achieved. In general, C_{sb} increases with source strength, scan time and lower noise, as would be expected.

Returning to equation V, it is obvious that C_1 involves the source strength and the time of scan while C_2 involves both the "noise" and source strength. If the source of noise is mostly background, C_2 diminishes as C_1 increases. If the noise is mostly leakage, C_2 remains constant. Figure 2 shows that the capacity varies only slightly with C_2 for two representative values of A and C_1, while Fig. 3 shows for two representative values of C_2 and A that the information capacity varies with the logarithm of C_1 as expected. It is to be particularly noted that the capacity varies only over a range of 5 to 1 for an enormous range of the constants and the maximum value presented is 15.5 bits per square inch.

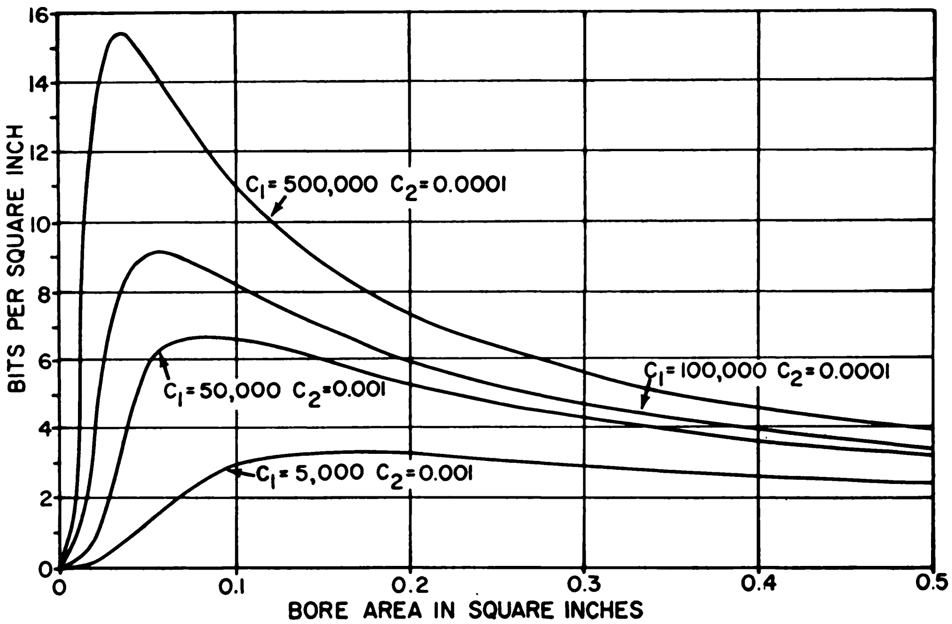

Fig. 1. Information capacity of a Single Bore Collimator as a function of bore area for several values of signal and noise.

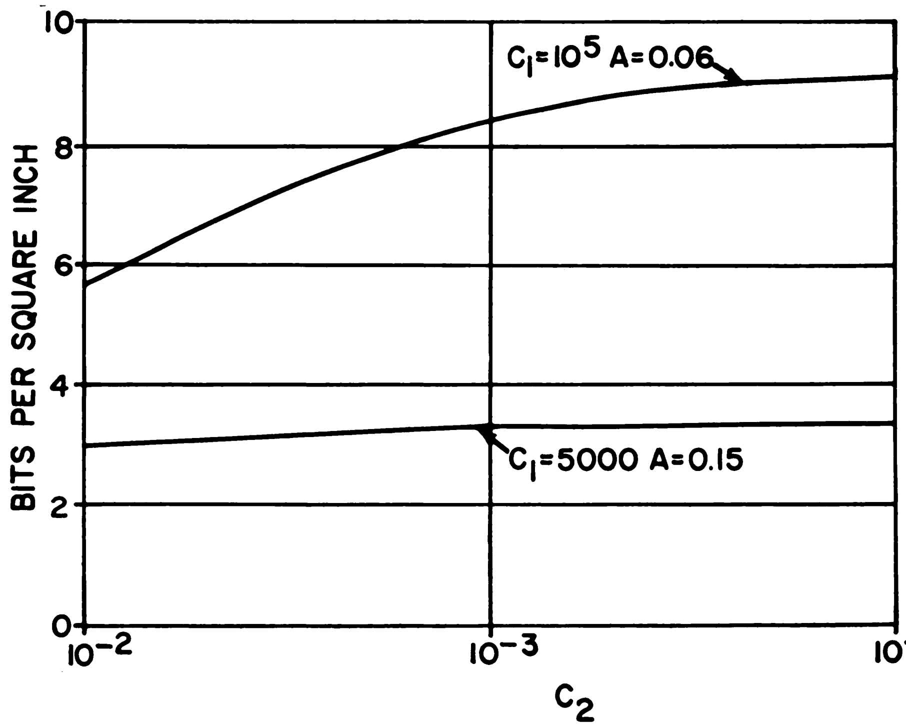

Fig. 2. Information capacity of a Single Bore Collimator as a function of noise for several values of signal and bore area.

If we had used K = 1 (optimum coding) this would only increase to 18.1 bits per square inch. The problem that remains is to evaluate the constants for typical collimators.

SINGLE BORE COLLIMATOR

Consider now a straight bore collimator as shown in Fig. 4 with a typical point source response shown in Fig. 5. Making a straight line approximation and integrating over the figure of revolution, we find for detected counts

$$S^* = \frac{\epsilon\sigma\pi\, d^4\tau}{64l^2}\left[\frac{2l^2}{t^2} - \frac{2l}{t} + 1\right] \tag{VI}$$

where ϵ is the efficiency of detection. The width at half-max is $\lambda = dl/t$ and the projected area $A_p \cong \dfrac{\pi\, d^2\, l^2}{4t^2}$.

For most single bore collimators, the response changes little (< 10%) with l around l = 2t and we may approximate further by

$$l^{-2}\left(\frac{2l^2}{t^2} - \frac{2l}{t} + 1\right) \cong \frac{5}{4t^2} \,.$$

Thus,

$$S^* = \frac{5\epsilon\sigma\pi\, d^4\, \tau}{(4)(64)\, t^2}$$

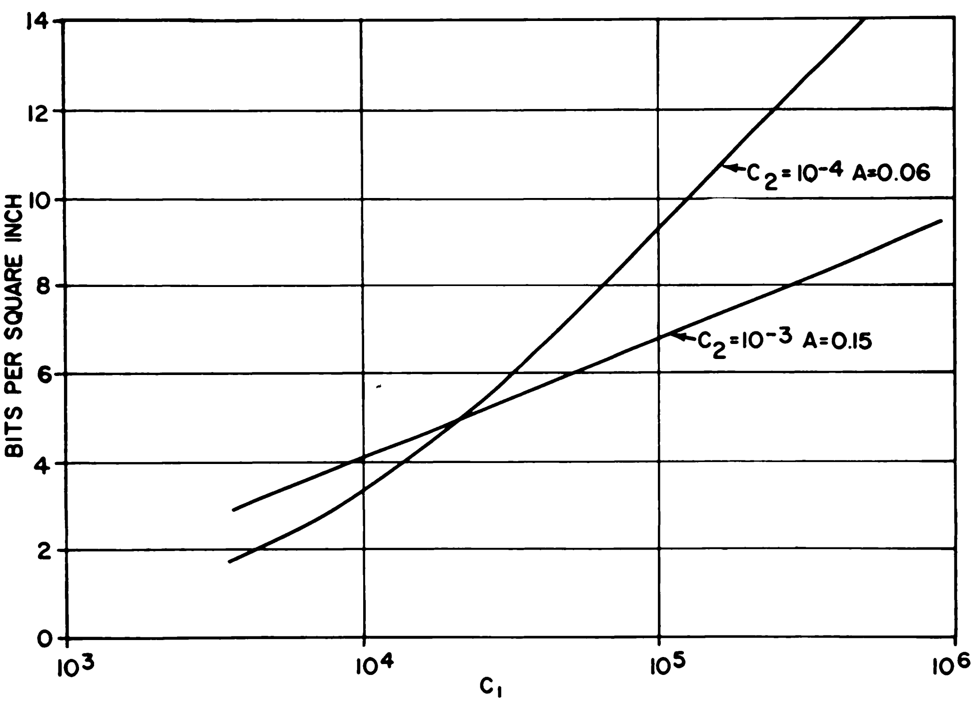

Fig. 3. Information capacity of a Single Bore Collimator as a function of signal for several values of bore area and noise.

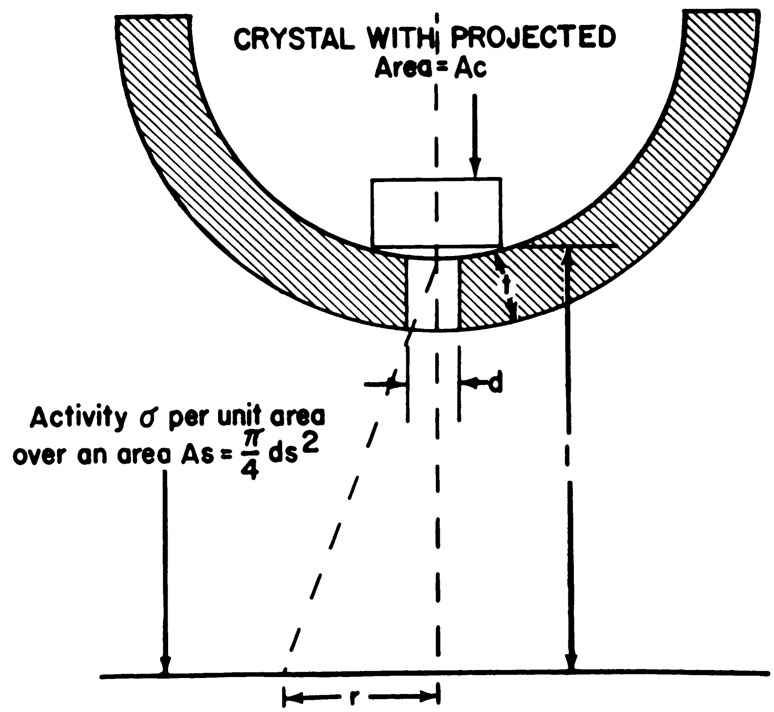

Fig. 4. Geometric relations assumed for a Single Bore Collimator viewing a planar distribution of activity.

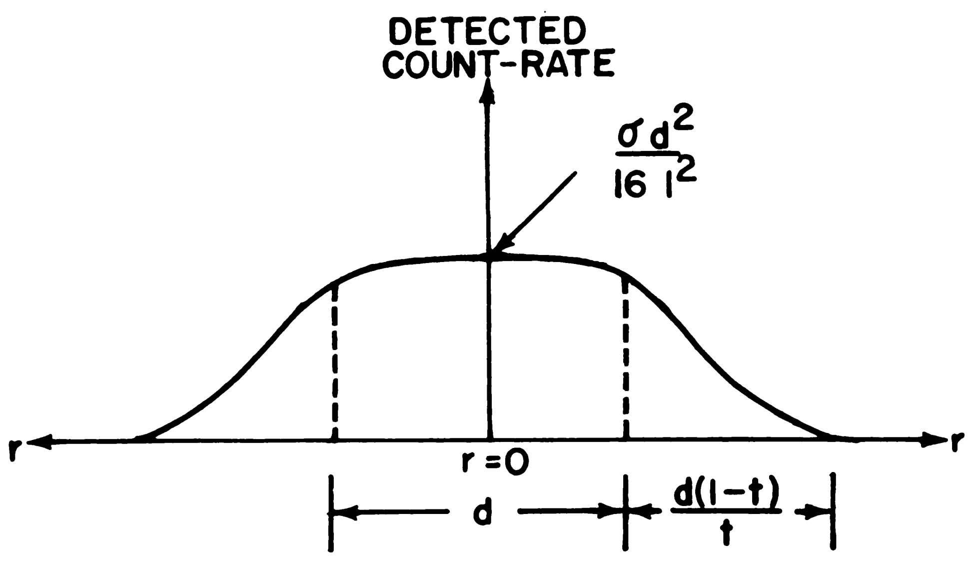

Fig. 5. Approximate count-rate profile for a point source of strength σ moved across the plane shown in Fig. 4.

Since $\frac{\pi}{4} d^2 = A$ we have then, $S^* = \frac{5\epsilon\sigma}{16\pi\, t^2} A^2 \tau$ (VII)

from which $K_1 = \frac{5\epsilon}{16\, \pi t^2}$ (VIIA)

Figure 6 shows observed count rates of various sized collimators as a function of the distance l of a planar source of activity 1.41 μc/sq inch of ^{131}I (367 kev) from a 2 × 2 inch crystal detector with a 50 kev window. This count rate is constant as expected. After correcting for background and leakage, calculations involving the known bore size led to values of K_1 that agree within 5 percent of those calculated from expression VII A with $\epsilon = 50$ per cent. This was also true of tapered bores when a bore diameter equal to the average bore diameter plus 10 per cent was used. This value of efficiency is quite reasonable for the circumstances.

Scatter was checked by observing the count rate over a 2 inch diameter empty hole in the plane of activity (now made 1 inch thick) and over a plastic plug of the same size immersed in the same medium. While no difference was observed in this case, such may not be true at lower energies. For the more practical examples discussed later we shall assume $K_2 \cong 0$.

Regarding leakage and collimator design, this is a much more difficult problem than first appears. Measurement of what might be called "leakage", however, is simply accomplished by observing the count rate when aiming the collimator at a large "hole" in a thick slab of activity. The results are generally much higher than expected for straight leakage through the lead shielding because of scatter at the edges of the collimator. Thus, analyzer window width, small percentage of higher energy gamma rays in the source, area of field activity, bore diameter and length, etc., all determine the apparent "leakage". Most single-bore collimators are about 3 inches long, which for ^{131}I radiation amounts to 46 half-value layers or a reduction of about 10^{-14} in linearly transmitted intensity. One can easily conclude that most collimators are designed from geometrical considerations only and made of thick lead to possibly reduce natural background and the influence of extraneous room sources and contamination. The geometrical considerations must obviously be a compromise between producing a well defined aperture (at various depths) and preserving a reasonably high count rate. A more exact calculation of these geometrical considerations may be made by referring to Fig. 4 and letting r be the distance from the center of a circularly symmetric plane are ($A_s = \frac{\pi}{4} d_s^2$) of activity σ per unit area to any point in the plane. The leakage count rate then becomes

$$R = \frac{\epsilon\sigma\, Ace^{-\mu t}}{2} \int_0^{d_s/2} \frac{rdr}{r^2 + l^2}$$

$$R = \frac{\epsilon\sigma\, Ace^{-\mu t}}{4} \log \left(1 + \frac{A_s}{\pi l^2}\right) \qquad \text{(VIII)}$$

Where μ is the linear absorption coefficient of the shielding and A_c is the effective cross sectional area of the crystal. For the case $A_s/\pi l^2 < 1$, we may approximate

$$R \cong \frac{\epsilon\sigma A_c A_s e^{-\mu t}}{16\pi t^2}$$

since S^*/τ is the detected count rate R^1 when viewing the plane we see

$$\frac{R}{R^1} = \frac{K_3}{K_1 A^2} \cong \frac{AcAs}{5A^2}\Sigma^{\mu t} \quad \text{or,}$$

$\frac{K_3}{K_1} \cong \frac{1}{5}\, AcAs\, \Sigma^{\mu t} \approx 10^{-16}$ for a typical 4 inch collimator considering leakage only.

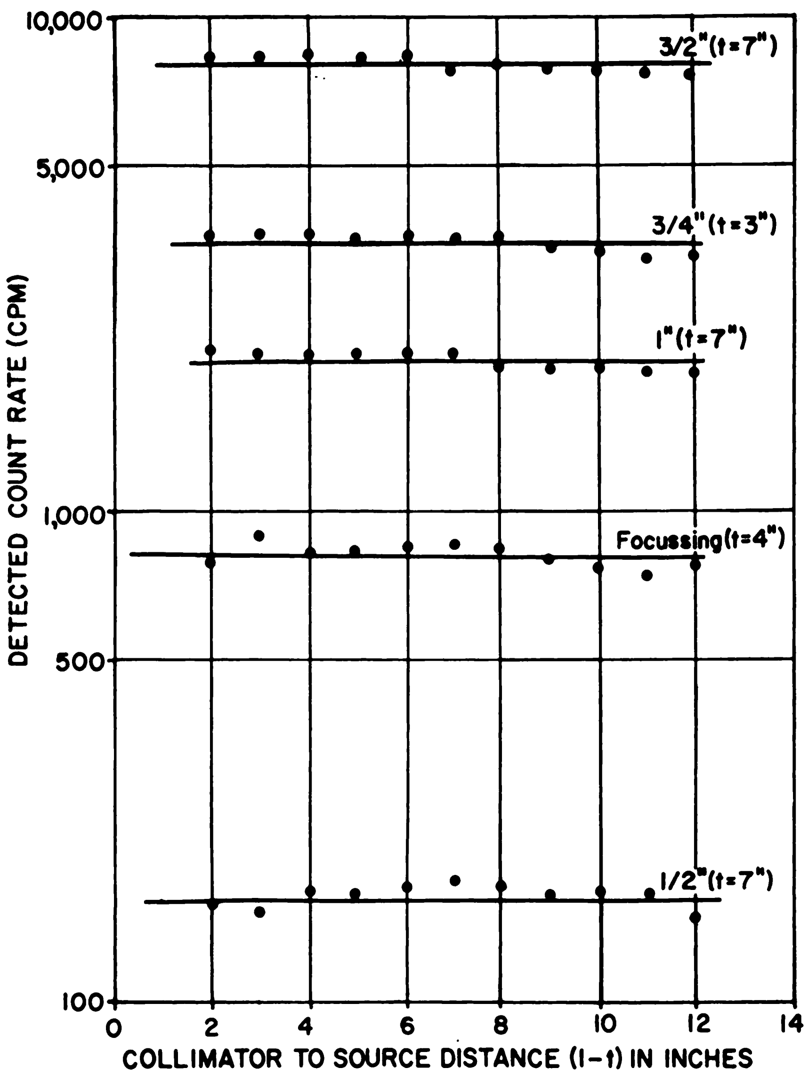

Fig. 6. Observed count-rate as a function of the distance (l − t) to a planar distribution of activity for various diameter single bore collimators. The source is ^{131}I with a density of 1.41 μc/sq inch.

Now, measurements on a typical 4 inch single bore collimator with a 2×2 inch crystal and a 50 kev window viewing a liver of 80 square inches containing an activity of 8 μc per square inch of ^{131}I shows negligible scatter and an R_b of 60 cpm of which 40 is due to background and 20 due to edgescatter. Since $K_1 = 3.2 \times 10^{-3}$ and $\sigma = 1.8 \times 10^{-7}$/min/square inch, we see $K_3 / K_1 \cong 4 \times 10^{-4}$. While this ratio is small compared to R_b/σ it is very large compared to the leakage effect calculated earlier.

Under the above circumstances and assuming for a typical liver scan that $T = ⅛$ min/sq inch and that we are observing at $l = 2t$ where $K_4 = 4$, we obtain $C_1 = 4800$ and $C_2 = 1.1 \times 10^{-3}$.

Using the appropriate curves in Fig. 1, we see that the capacity rises to a peak of about 3.4 bits per sq inch for a collimator area of 0.12 sq inches (0.4 inch diameter). Further, the variation with A is slow except at very small values of A. This variation is bourne out in practice where not much apparent difference in scans is obtained for large ranges of A until A becomes small. The use of two opposing single bores doubles C_1 and halves C_2 which only raises the maximum capacity to 4.4 bits per sq inch. Since a factor of 2 in C_2 produces little change in the capacity, one could consider a collimator constructed of holes in two slabs of lead an outside distance t apart to preserve the geometry. One would need a total of only 2.6 cm of lead before affecting C_2 by more than 2. Such a design might also reduce edgescatter as well as the volume of lead.

Considering the fact that the capacity varies slowly with area above 0.08 sq inches in the above example, one can rightfully ask the advantages of small versus large bore collimators over this range. The original formulation of information capacity did not weigh any particular resolution but merely asked the ability of the system to handle all sizes of signals. It is obvious that a large bore will be able to detect large defects well because of its high count rate, but miss the ones smaller than its projected area. Likewise a small bore will be able to find many more small defects but less well because of its lower count rate. It follows that the clinician must weigh the size by his clinical objectives and that for certain diseases, it may well be very desirable to work with the smallest bore possible even to the extent of having a reduced information capacity. The disadvantages, however, are twofold. Firstly, the smaller bore will see more "defects" including those variations in the normal organ and a new set of normals will have to be established. Secondly, the count rate may be so low (even with the same information capacity) that the method of presentation will affect the actual overall capacity. The above formulation assumes that the density of counts is great enough that the eye can integrate over an area and sense an average density. If the spots are too far apart, this process becomes difficult and the K value drops considerably. Partial compensation can be accomplished by minification of the image but the ability of the eye to resolve two adjacent defects then becomes the limiting factor. If one then proposes a superposition of less dense dots to produce an average density over an area A_p, recording compression usually enters and one must resort to contrast enhancement. This approach, however, has the pitfall that film "noise" becomes the limiting factor. While this particular problem has not yet been satisfactorily solved, clinical results to date indicate that a ⅜ to ½ inch diameter straight bore is acceptable for most problems, and that in some cases little is lost by going up to ¾ inch. In most cases of controversy, the method of presentation is usually at fault.

FOCUSSING COLLIMATORS

A focussing collimator is simply an aggregate of single bore collimators all arranged to look at the same plane. While each single bore has a much smaller information content, the combined action of all is, of course, higher. One may look upon these collimators simply as devices that increase the optical efficiency where now the counts detected by the crystal are $S^* = nK_1 \sigma A^2 \tau$ for n holes looking at the same point in space. The actual aperture area, however, is somewhat larger than A_p for each hole of area A, since the outside holes are at an angle θ with respect to the central hole and the planar diameter is the projected diameter divided by Cos θ. This effect will lower the information capacity as graphed in Figs. 1-3 by about 25 percent, since A_p is sensitive to the square of the diameter.

Consider now a typical 19 hole focussing collimator, 4 inches thick with a total measured background of 110 cpm. For the same scan conditions as for the single bore collimator analyzed previously, we have

$$C_1 = K_1 K_4 K^2 \sigma T^1 = 9.1 \times 10^4 \text{ and } C_2 = R_b / nK_1 = 1.0 \times 10^{-4} .$$

Interpolating from the curves shown in Fig. 1, we see that there is a pronounced peak in the capacity of 9.2 bits/sq inch at A = 0.06 sq inches (diameter of 0.28 inches). The actual capacity due to angulation of the bores is about 7 bits per square inch, which is only twice that of the single bore collimator. It is interesting that a commercially available collimator of this size used clinically for liver and kidney scans has tapered holes that have an effective diameter of 0.25 inches illustrating that long practice and experience generally result in optimum design. While the capacity of this focussing collimator is larger than an optimum single bore, a factor of two at these low values is generally considered unimportant and several investigators have reported their opinion that, clinically, the two produce similar results. It does remain, however, that the count rate is much higher and this probably makes crude presentation techniques more acceptable and easier to perceive as mentioned.

Focussing collimators, however, do have one disadvantage and that is for off-plane defects the capacity drops considerably. For example, consider attempting to see a small defect so far off the focal plane that only one collimator hole detects it. The remaining n-1 holes are still recording signals from the plane which signals must be considered noise since they do not originate from the same point in space. This would lower considerably the information capacity for this particular plane. Rather than pursuing this approach, we may note that equation IV would hold for any S^* (even that originating in a volume) provided a unique A_p could be specified. Since S^* and N^* are independent of l, we can now calculate an average information content over the range of l encountered in a medium.

$$\overline{C} = \text{Avg.}\left\{\frac{1}{A_p}\log_2\left[1+\left(\frac{K^2\ (S^*)^2}{S^*+N^*}\right)^{\frac{1}{2}}\right]\right\}\begin{matrix} l = l_2 \\ l = l_1 \end{matrix}$$

$$\overline{C} = (Const.)\ \text{Avg.}\left(\frac{1}{A_p}\right)_{l_1}^{l_2} = (Const.)\left(\overline{\frac{1}{A_p}}\right) \qquad \text{(IX)}$$

Consider a straight bore collimator where $A_p = \pi\, d^2 l^2/4t^2$.

Now
$$\left(\overline{\frac{1}{A_p}}\right) = \frac{1}{l_2 - l_1}\left(\frac{4t^2}{\pi d^2}\right)\int_{l_1}^{l_2}\frac{dl}{l^2}$$

$$\left(\overline{\frac{1}{A_p}}\right) = \frac{4t^2}{\pi d^2\, l_1\, l_2}$$

For a medium t thick starting a distance t/2 below the bottom of the collimator, we find

$$\left(\overline{\frac{1}{A_p}}\right) = \frac{1}{\pi d^2}$$

which is fortuitously the same as that used previously for the straight bore looking at a plane at a distance t from the bottom of the collimator. If we had used a medium 2t thick starting at the bottom of the collimator, then $\left(\overline{\frac{1}{A_p}}\right) = \frac{2}{\pi d^2}$ which would show a higher capacity than that for the single bore mentioned. This higher capacity results since $1/A_p$ preferentially weights smaller areas which occur at shorter distances for the straight bore. Thus, a straight bore collimator looking at a volume t thick starting a distance t/2 from the collimator will have an average information capacity the same as if it were looking at a planar distribution a distance t away.

If we now look at the geometry for a focussing collimator shown in Fig. 7 and assume that the aperture diameter diminishes linearly from d + 2t Tan θ^1 to 2d/Cos θ^1, we ultimately find for a medium of thickness t starting t/2 away from the bottom of the collimator

$$\left(\overline{\frac{1}{A_p}}\right) = \frac{4}{\pi}\left(\frac{d}{Cos\ \theta^1} + \frac{d + 2t\ Tan\ \theta^1}{2}\right)^{-1}\left(\frac{2d}{Cos\ \theta^1}\right)^{-1} \simeq$$

$$\frac{2\ Cos\ \theta^1}{\pi\ dt\ Tan\ \theta^1} \text{ for } \frac{d}{Cos\ \theta^1} + \frac{d}{2} << t\ Tan\ \theta^1$$

which is usually the case. For most focussing collimators, $\theta^1 \approx 30°$ and t $\approx$ 16d, so

$$\left(\overline{\frac{1}{A_p}}\right) \cong \frac{1}{8\pi\, d^2}$$

This result states that the average information capacity for a focussing collimator looking at a volume is about ⅛ that of the capacity when looking at the focal plane. This obviously arises because of the weighting of the large effective aperture areas on both sides of the focal plane. Thus, we have for the practical examples sited previously, average information capacities of 4.4 bits per square inch for the straight bores and 1.1 bits per square inch for the focussing collimator when viewing defects in a large medium.

In practice, this large difference probably does not exist since (a) most defects of clinical interest are located near the focal plane and (b) the observation of one defect reduces the clinical importance of observing additional ones. If one knows the plane of interest (*i.e.* as in most brain scans), the focussing col-

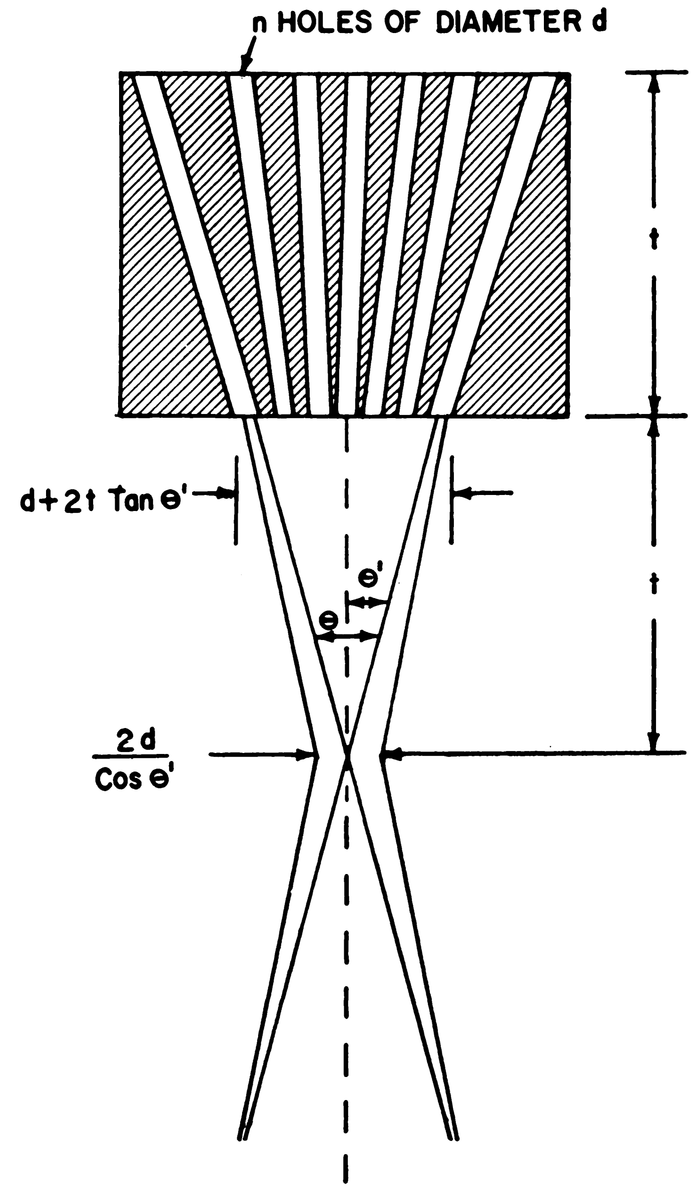

Fig. 7. Geometic relations assumed for Focussing Collimators. The outside lines are the envelope for A_p as a function of distance from the collimator.

limator is superior by the factor of 2 mentioned previously. A typical 199 hole focussing collimator designed for brain scans gave a calculated capacity of 10 bits/sq inches for focal plane activity. This magnitude of improvement is becoming significant.

HONEYCOMB COLLIMATORS

A honeycomb collimator is simply an aggregate of straight bore collimators with a system that detects and presents in its proper place the activity seen by each "hole." The Bender-Blau "autofluoroscope" (13, pg. 151) is an example of this type collimator. There is no scanning motion involved, which means that each hole is recording for the complete scan time T_s.

$$\text{Now } S^* = K_1 \sigma A^2 T_s \text{ and } N^* = (K_2 \sigma A^2 + K_3 \sigma + N_b) T_s .$$

This formulation neglects the change in efficiency and background as A becomes smaller. In other words, it assumes that the volume of the detecting crystal remains constant. This assumption should not change the conclusions appreciably. Letting C_h represent the capacity of a honeycomb collimator, substitution of the above produces

$$C_h = \frac{1}{K_4 A} \log_2 \left[1 + \left(\frac{C_1^1 A^4}{A^2 + C_2} \right)^{\frac{1}{2}} \right] \tag{X}$$

where $C_1^1 = \frac{K^2 K_1^2 \sigma T_s}{K_1 + K_2}$ and $C_2 = (K_3 \sigma + N_b)/(K_1 + K_2) \sigma$

as before. Assume now that we are looking at the same field of activity as in the previous examples with a honeycomb collimator 4 inches thick. Also assume as before that $K_4 = 4$, $K = \frac{1}{4}$, $T_s = 27$ minutes, $N_b = 100$ cpm in each detector, and that volume scatter (K_2) and leakage (K_3) are negligible. Thus,

$$C_1^1 = 10^5 \text{ and } C_2 = 1.7 \times 10^{-3} .$$

Since the equation (X) is slightly different than (V), due to the lack of scanning motion, a different computing program was written for this and the results are plotted in Fig. 8 for $K_4 = 4$, $K = \frac{1}{4}$, and various values of C_1^1 and C_2. For the above examples, C_h is a maximum of about 22 bits per square inch at $A = .02$ sq inches. This corresponds to holes of 0.16 inch diameter. No extensive experience with this type collimator is available to check these results. It is to be noted that a tacit assumption is that the honeycomb is so designed that each A_p is contiguous with surrounding A_p's and that there are no blank spots. This assumption states that for the plane of observation at $l = 2t$, the thickness of shielding between holes is equal to the hole diameter. The equation also assumes that the signal detected by each hole is finally presented over an area A_p. This can be accomplished by placing the detecting plane as far above the honeycomb as the observed plane is below it. In the usual honeycomb detecting system, sidescatter of the absorbed gamma rays generally affects the apparent resolution when the detecting crystals are close together and are not shielded from each other. This effect has been estimated to be about 10 per cent which amounts to a 25 per cent reduction in the calculated capacity. Partial compensation can be accomplished by constructing the detecting plane closer to the crystal so that scatter essentially increases the recorded area to A_p.

PINHOLE CAMERA

For the pinhole camera (14) as shown in Fig. 9, each point source irradiates a circle of area $A_p = \pi D^2/4$. This is the minimum resolvable area. Now each point in A_p sees an activity $\sigma\epsilon\ (D^1)^2/16l^2$, so that for the usual case of $l = 2t$, the total activity seen in A_p is $S^* = \sigma\epsilon\pi T_s d^4/16t^2$. Letting $A = \pi d^2/4$, then $A_p = 4A$, and $S^* = K_1\ \sigma\ A^2\ T_s$, where

$$K_1 = \frac{\epsilon}{\pi t^2}.$$

In this style camera, the detector is a continuous plane (or crystal) and the background is a constant rate per unit area rather than per A_p as before. Hence, neglecting volume scatter, $N^* = 4\ (N_b + K_3\sigma)\ A\ T_s$. Letting C_p represent the capacity of a pinhole collimator, we find

$$C_p = \frac{1}{4A} \log_2 \left[1 + \left(\frac{C_1 A^3}{A + C_2}\right)^{\frac{1}{2}}\right] \text{ bits/unit area, where } C_1 = K^2 K_1 \sigma T_s$$

and $C_2 = 4(N_b + K_3\sigma)/K_1\sigma$. This equation is plotted in Fig. 10 for $K = \frac{1}{4}$, and various values of C_1 and C_2.

In order to compare with other style collimators assume the same activity per unit area as before, $T_s = 27$ minutes and $t = 4$ inches. Since the previous data

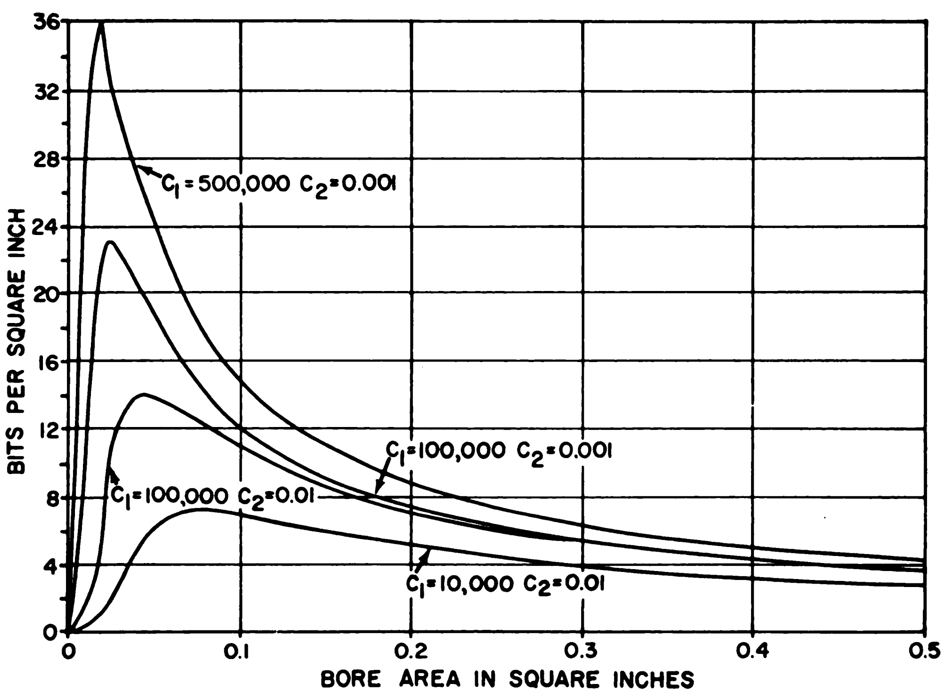

Fig. 8. Information capacity of a Honeycomb Collimator as a function of the bore area of each hole for several values of signal and noise.

for background ($N + K_3\sigma_b = 100$ cpm) was for a 2 inch diameter crystal, we have for this case $(N_b + K_3\sigma) = \frac{100}{\pi} \simeq 30$ cpm/sq inch. Now most designs of this style camera use a thinner detector, so we may assume $\epsilon = 0.2$ instead of 0.5 as before. These assumptions lead to $C_1 = 1.2 \times 10^5$ and $C_2 = 2 \times 10^{-4}$.

From the given curves, these data produce a maximum capacity of 43 bits/sq inch at A = .08 sq inches. This is the area of the actual pinhole and it is suspected that in practice it could be physically smaller, since edge leakage would tend to make it appear slightly larger. The information capacity for the pinhole is higher than that for the honeycomb simply because of the shielding present between holes in the honeycomb.

It is interesting that if we assume that the previously discussed honeycomb collimator is constructed of long crystals whose area changes as the holes change, we achieve exactly the same formulation as above. The only difference is that C_1 is 1/3 as great and C_2 is three times larger than the pinhole because of the different formulation for K_1. This difference arises because of the poorer geometrical efficiency of straight bores as mentioned. Thus, overlooking practical difficulties, (relative leakage, crystal efficiency, etc.) the pinhole has the possibility of a higher information capacity than the honeycomb.

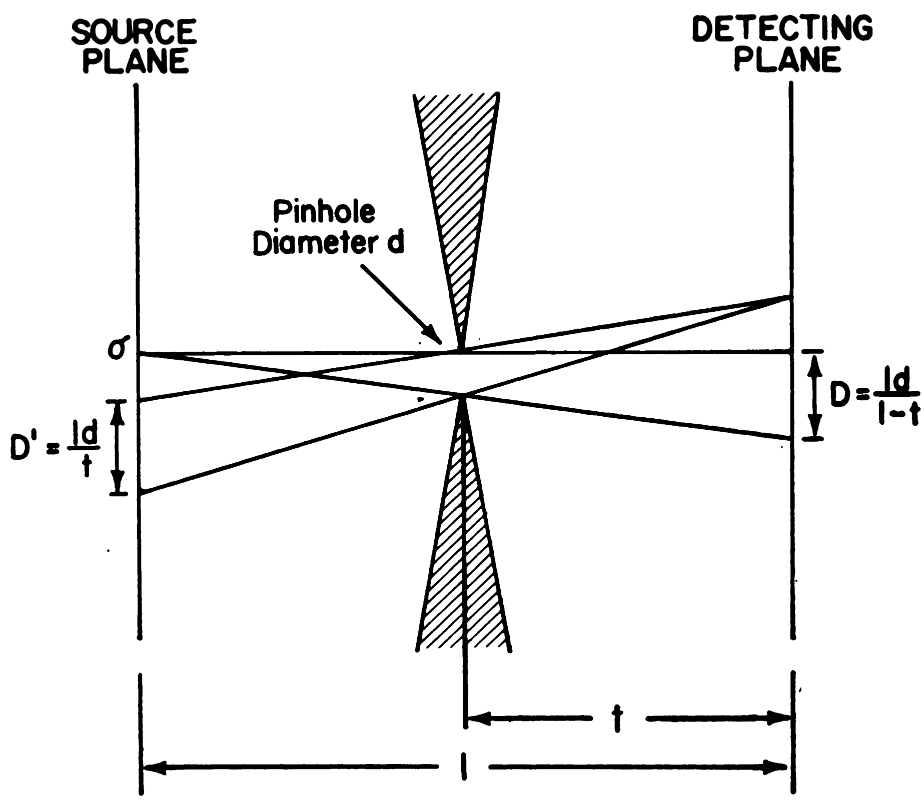

Fig. 9. Geometric relations assumed for a Pinhole camera.

SCAN TIME AND SOURCE STRENGTH CONSIDERATIONS

Consideration of the previous formulations will show that for a fixed source strength σ and resolution A_p, the time of scan for a given collimator is related to the information capacity by

$$\frac{T_1}{T_2} = 2^{A_p (C_1 - C_2)}$$

where C_1 and C_2 are the information capacities for times T_1 and T_2. Thus, the optimum honeycomb discussed earlier will produce the same information capacity as an optimum straight bore if a scan time of 9 minutes is used instead of 27 minutes. With a scan time of 12 minutes, the capacity will be the same as for the optimum 19 hole focussing collimator scanning the same plane for 27 minutes. It is to be emphasized, however, that the resolutions are different for all three cases even though the capacities are equal. For the optimum straight bore to produce the same capacity as the optimum 19 hole focussing collimator scanning a plane, the time of scan must be increased by about a factor of 3. The same considerations will hole for variations in source strength for constant time provided the noise is due to background only. If leakage is present, the logarithmic term will not change as rapidly as the change in source strength would imply.

CONCLUSIONS

1. Scintiscans have an information capacity quite low compared with conventional visual presentations (*i.e.* films, television, etc.).

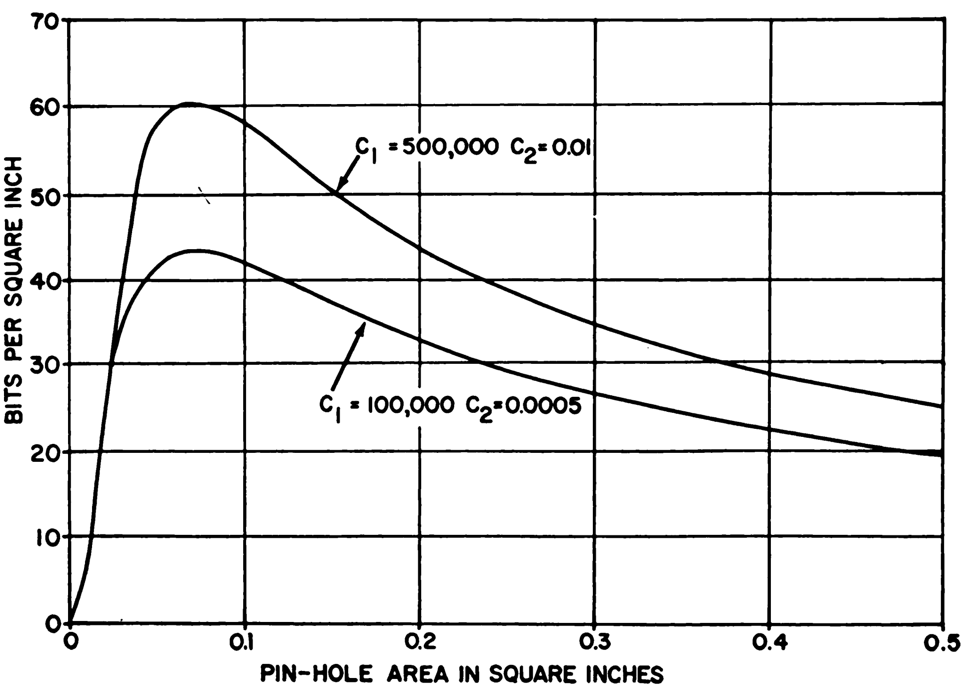

Fig. 10. Information capacity as a function of Pinhole area for several values of signal and noise.

2. All styles of collimators are characterized by having an optimum bore size—above which and particularly below which the information capacity diminishes.
3. The straight bore collimator is slightly superior to the focussing collimator when viewing distributions in depth; the focussing collimator is superior when viewing reasonably well located planar distributions.
4. Honeycomb and pinhole collimators have a much greater information capacity than mechanically scanned collimators simply because there are more detectors and each detector is recording during the total scan time.
5. The visual presentation method is most important and cruder methods, such as "dot-on-paper", reduce the available capacity simply due to the characteristics of the eye. The presentation system must be carefully designed to achieve the calculated maximum capacities.
6. Information capacity calculations do not weight any particular size of signal areas. Minimum bore size should be selected by a compromise between clinical desires and (2) and (5) above.
7. The information capacity of a given scintiscanning system changes roughly as the logarithm of the product of the specific activity and the time of scan.

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