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Determination of Organ Volumes by Scintillation Scanning[1,2]

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Organ-localizing radioactively labeled materials have provided a means for locating organ contours by use of external scanning. Less attention has been paid to the problem of determining organ volumes. By use of gamma-emitting radioisotopes, and scintillation scanning, data can be obtained which allow the calculation of such volumes. In cases in which the amount of a therapeutic agent to be administered depends on the organ volume, it is desirable to have an estimate of this quantity. In addition, there are other uses of these data, as in following the postnatal development of an organ, and in the comparison of organ sizes between species. We present here a discussion of the theory of estimating such volumes, and illustrate the general discussion by some specific examples.

THEORY

Following the scanning of an organ on anteroposterior and lateral views, the volume of the organ would be given (dimensionally) by an equation of the following form:

$$V = k(A_a \, . \, A_l)^{\frac{3}{4}} \qquad (1)$$

where A_a is the area of the anteroposterior scan, A_l is the area of the lateral scan, and k is a constant. This formulation holds precisely for several types of figures. Simple algebraic proof is presented for 3 cases.

(A) Sphere. The area of the anteroposterior scan of a sphere is equal to that of a circle, and so is the lateral scan. Hence,

$$V = k(\pi r^2 \, . \, \pi r^2)^{\frac{3}{4}} \qquad (2)$$

$$V = k\pi^{\frac{3}{2}} \, r^3 = cr^3 \qquad (3)$$

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with c a constant (which is, of course, equal to $\frac{4}{3}\pi$).

(B) Cube. The areas of both the anteroposterior and lateral scans of a cube are equal to L^2, with L the length of one side. Therefore:

$$V = k(L^2 \; . \; L^2)^{\frac{3}{4}} \tag{4}$$

$$V = kL^3 \tag{5}$$

In this case, k would equal 1.

(C) A cylinder, such that: H = q.R (6)

That is, with the height expressible as a simple multipicand of the radius.

The anteroposterior and lateral scans of a cylinder reveal a rectangle with height H and width 2R, R is the radius). Hence, we can write:

$$V = (2RH \; . \; 2RH)^{\frac{3}{4}} \tag{7}$$

substituting for H from equation (6) yields:

$$V = k(2qR^2 \; . \; 2qR^2)^{\frac{3}{4}} \tag{8}$$

$$V = b \; . \; R^3 \tag{9}$$

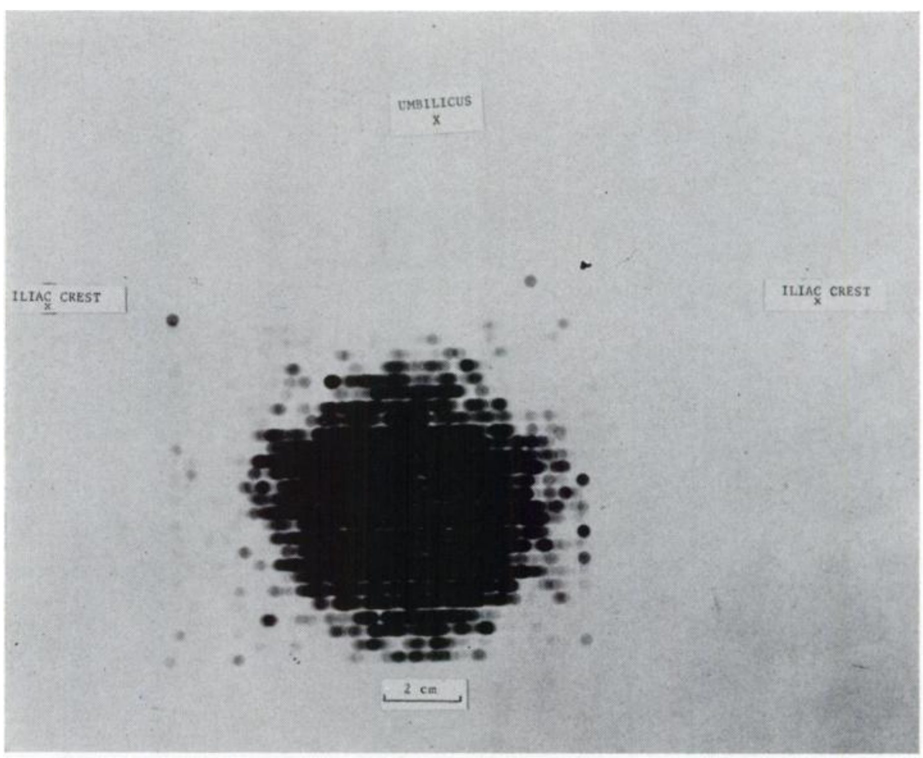

Fig. 1. Anteroposterior bladder scan of an 11 year old girl, following use of Hg^{203}-chlormerodrin.

with b a new constant. Such an argument also holds for a cone in which the height is a simple multipicand of the radius.

The volume of more complex figures is perhaps most simply approached by measuring the various linear dimensions and constructing a suitable model. Again, by dimensional analysis, the general equation would be of the form:

$$V = K(L.W.H.) \tag{10}$$

where K is a constant and L is the length, W is width, and H is height. This, of course, holds exactly for only certain figures. It must be recognized that when figures *in vivo* deviate from precisely defined shapes, estimation of their volumes becomes less exact.

In some cases it may be possible to relate the observed scan with a well defined solid figure whose volume can be calculated. For example, the non-dis-

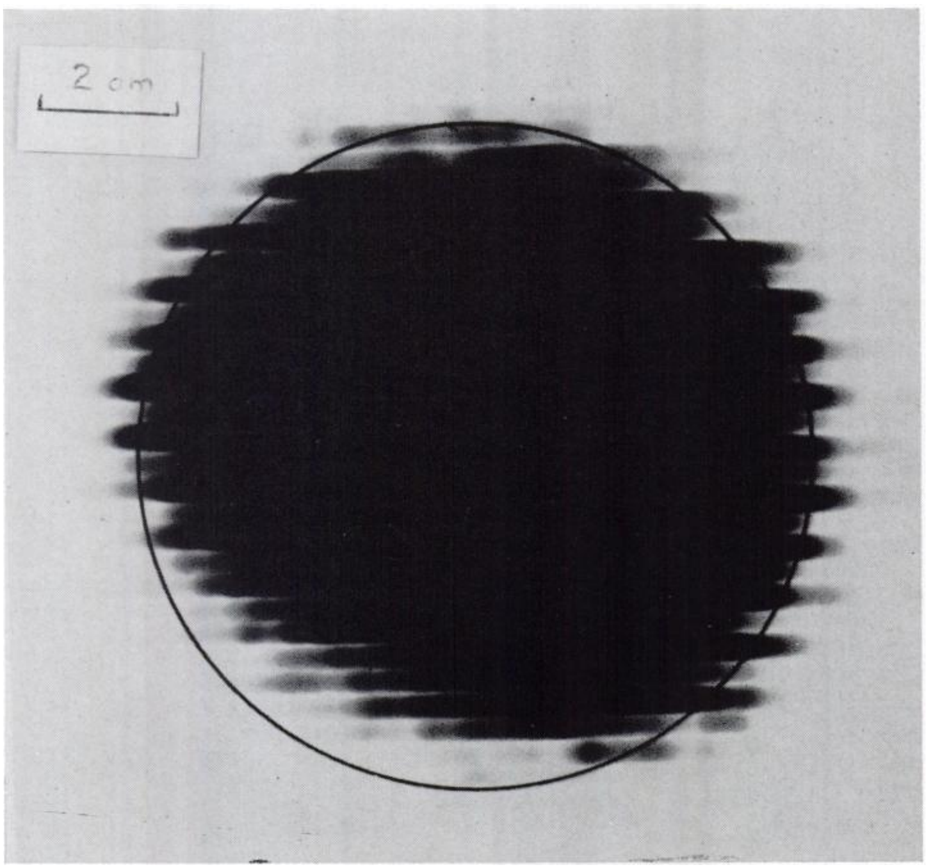

Fig. 2. Right lateral scan of the bladder of a 20 year old man, utilizing Hg^{203}-chlormerodrin. The outline drawn about the scan is that of a circle. Observe that due to external compression, a spherical segment of one base appears to have been cut off the spherical bladder.

tended human bladder has been observed on scans to closely approximate a spherical segment of two bases. The volume of such a figure is given by:

$$V = \frac{h\pi}{6}(3r_1^2 + 3r_2^2 + h^2) \qquad (11)$$

where r_1 and r_2 are the radii of the bases and h is the height.

An additional approach is possible by recognizing that the volume of some figures can be described by simple integrals once the equations for their contour or cross-sectional area are known. There are three principal techniques (1).

(a) If the outline of the figure can be described by y = f(x), and if the figure is rotated about the x-axis between the coordinates x = c and x = d, the volume is given by:

$$V = \pi \int_{x=c}^{x=d} y^2 dx \qquad (12)$$

(b) An alternative to the above procedure is to divide the figure into multiple cylindrical shells. The volume is then given by:

$$V = 2\pi \int_{x=c}^{x=d} xy\, dx \qquad (13)$$

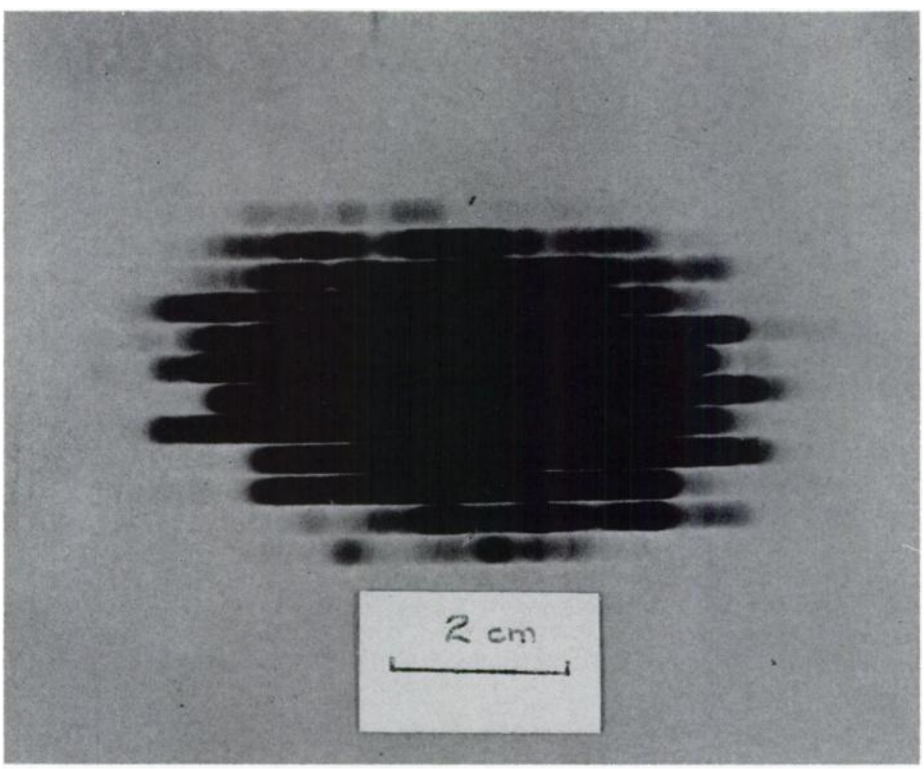

Fig. 3. Bladder scan of a 65 year old man using Hg^{203}-chlormerodrin. The scan resembles a spherical segment of two bases.

(c) For any solid figure for which the area of a cross section at a distance x from a fixed plane, can be expressed in terms of x, the volume can be obtained from:

$$V = \int_{x=c}^{x=d} A(x)dx \qquad (14)$$

where A(x) is the area in terms of x.

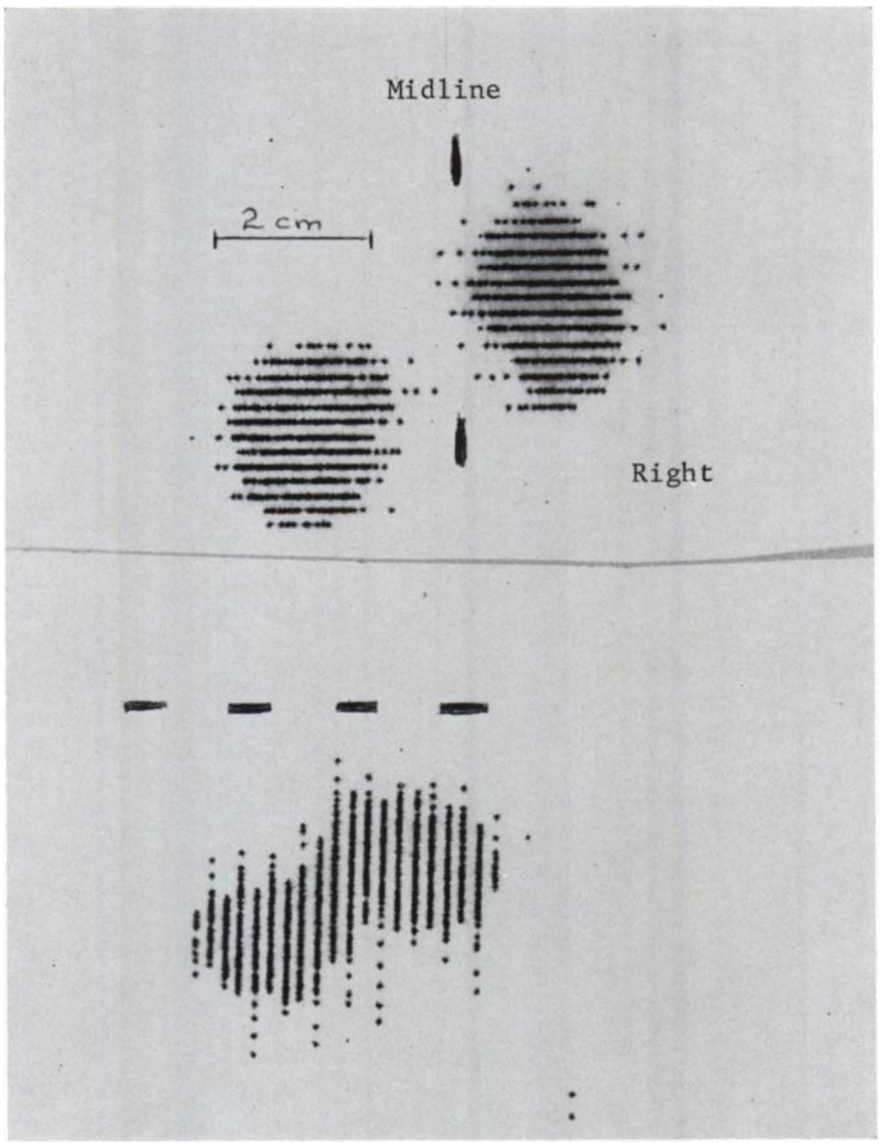

Fig. 4. Posteroanterior (top) and right lateral (bottom) scans of the kidneys of a 473 gram male chinchilla, using Hg^{203}-chlormerodrin.

METHODS

Human bladder scans were performed 2 hours after the intravenous (or deep intramuscular) injection of 50-100 microcuries of Hg^{203}-chlormerodrin. The scanning instrument was a Picker Magnascanner with large collimator (focal distance 7.0 cm) set just above the skin surface. Lines were traced 0.5 cm apart at a scanning speed of 20 cm/sec. Renal scans in adult chinchillas and rats were performed 2 hours after administering 50 microcuries of the radiolabeled chlormerodrin; good scans could also be obtained at only a fraction of this quantity of compound. Chinchillas were kindly provided by Mr. Helmut Gehmlich of the Gayla Chinchilla Ranch. For scanning the animals, the small collimator was employed and was set 5 cm away from the skin. The scanning speed was reduced

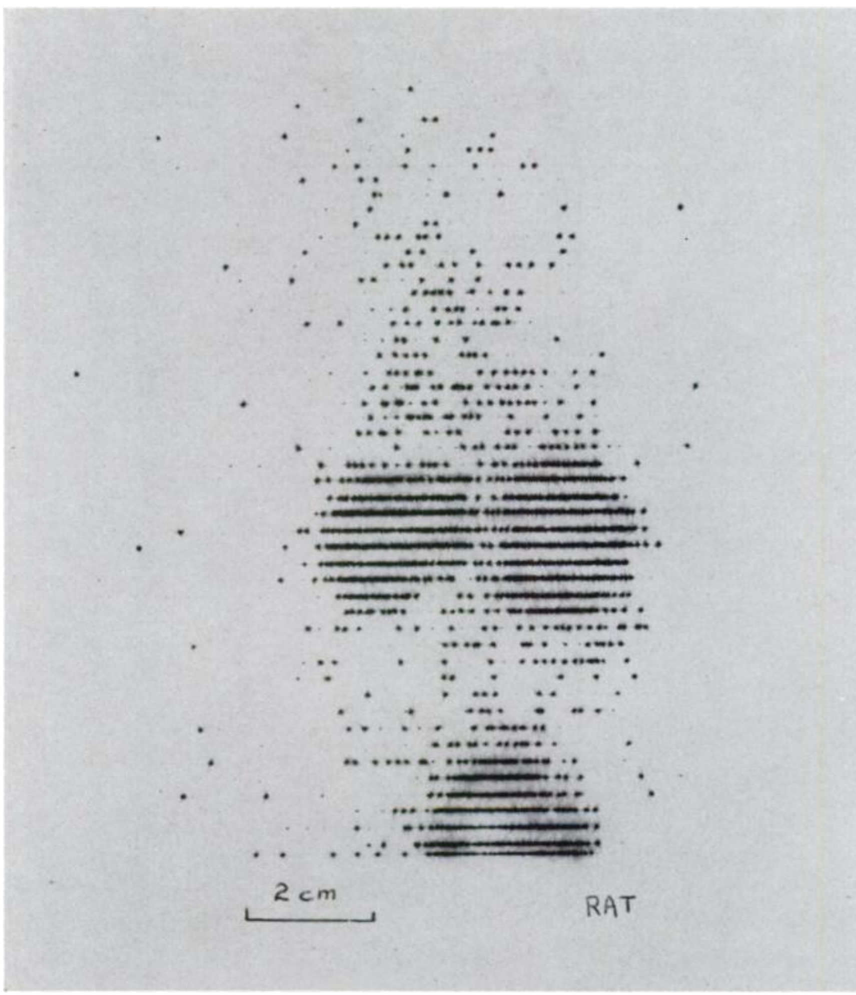

Fig. 5. Renal scan following administration of Hg^{203}-chlormerodrin to an adult male rat. The dome of the bladder can be visualized in addition to the kidneys.

to 15 cm/sec. and the lines were set at 0.25 cm apart. To clearly distinguish the organ under study from the background, it was essential to have as short a time constant as possible (so that counts were not averaged over a prolonged period and "spilled" from organ outline to background). We have used a ¼ or ⅛ second time constant. It was observed that when given subcutaneously, Hg^{203}-chlormerodrin was not efficiently absorbed from the depot site in the chinchilla within 3 hours. After intramuscular injection, there was still some retained activity at the injection site 3 hours later.

RESULTS AND DISCUSSION

Anteroposterior scans of the distended human bladder closely approximate a sphere (Fig. 1). If the organ were indeed a perfect sphere, the volume in cm^3 could be readily calculated from the radius (or from the area estimated by planimetry, by cutting out the scan and comparing its weight with that of a standard area, or by use of any one of a number of approximation techniques (2).

Bladder Volume in cm_3	*Radius of sphere in cm*
1,000	6.2
800	5.8
600	5.2
400	4.6
200	3.6
100	2.9
50	2.3

One pitfall is that the bladder may be compressed somewhat in the anteroposterior direction; hence the need for a lateral scan. Corrections can be made for such compression, and accurate urinary volumes determined. Figure 2 shows the lateral bladder scan of a young man. A circle has been drawn about the outline. The scan closely fits the outline, except at the posterior-inferior border, where a spherical segment of one base has been removed by compression. The volume of a spherical segment of one base is given by:

$$V = \frac{\pi h}{6} (3r^2 + h^2) \qquad (15)$$

where r is the radius of the segment and h is its height. Hence, the volume of the bladder pictured in Figure 2 is equal to that of a sphere minus the "missing" spherical segment.

The minimally filled urinary bladder often closely approximates a spherical segment of two bases (Fig. 3). The volume of such a figure can be calculated from equation (11). We have scanned such bladders in both the anterposterior and lateral directions before and after voiding. The voided volume of urine (original bladder volume-final bladder volume) generally agrees ($\pm$ 10 cm^3) with the quantity of urine actually collected.

Two hours following the injection of Hg^{203}-chlormerodrin in the chinchillas, a renal scan was performed (Fig. 4). It can be seen that the scans approximate an ellipsoid of revolution. The volume of such a figure can be calculated from equation (12) since the equation for an ellipse is:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ The volume of an ellipsoid turns out to be:

$$V = \frac{4\pi}{3} b^2 \alpha \tag{16}$$

where b is the minor semiaxis and a is the major semiaxis. The chinchilla kidneys are not perfect ellipsoids of revolution (there is an indentation at the hilus), there is a slight magnification factor involved at these machine settings, and a factor is needed for conversion of volume to weight. The final equation for calculating the weight of the chinchilla kidneys thus contains a correction factor. The weight of the left kidney illustrated was calculated to be 1.74 gm (actual weight 1.76) and that of the right kidney 1.64 gm (actual weight 1.73). Over a series of 5 chinchillas thus far, the estimate has been ± 8 per cent of the actual weight. It should be emphasized that for other machine settings the value of the correction factor will be different (and, hence, careful placement is essential each time). Renal scans can be performed on other animals as well, and such a scan in the rat is shown in Figure 5.

Attempts have previously been made to estimate the weight of the thyroid gland[3] on the basis of the anteroposterior scintillation scan. The thyroid is, of course, a three dimensional organ, and scans from front to back are inadequate to give information as to the depth of the gland. Hence, a lateral scan should be performed, as well as an anteroposterior scan, when an attempt is being made to evaluate the volume of the thyroid, or any other organ.

Estimation of the volume of internal organs is presently potentially possible for each of the following organs, by use of scintillation scanning.

Organ	*Technique*
Thyroid	Uptake of radioiodine.
Liver	Gold uptake, or radiolabeled rose bengal.
Spleen	Use of Cr^{51}-red cells which were then heat denatured.
Kidney	Mercury-labeled chlormerodrin or iodinated dye.
Bladder	Mercury-labeled chlormerodrin or other material that accumulates in the urine.
Pancreas	Se^{75}-selenomethionine.

It must be emphasized that the procedures discussed here for estimation of organ volumes are but a first approach, and refinement will be necessary. Already interesting variants are presenting themselves for analysis. For example, infarcts tend to resemble wedge-shaped regions whose volumes can be estimated by suitable equations. The ability to estimate the volume of functioning, and nonfunctioning, tissue may be a useful adjunct in both clinical and zoological studies (4, 5).

SUMMARY

A discussion of estimating organ volumes on the basis of anteroposterior and lateral scintillation scans is presented. The discussion is illustrated by the determination of the volume of the human bladder, and the weight of the chinchilla kidney. Potential uses of the techniques are pointed out.

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