Absorbed Fractions for Electrons and Beta Particles in Spheres of Various Sizes

Jeffry A. Siegel and M. G. Stabin

Cooper Hospital, University Medical Center, Department of Radiation Oncology, Camden, New Jersey; and Radiation Internal Dose Information Center, Medical Sciences Division, Oak Ridge Institute for Science and Education, Oak Ridge, Tennessee

The use of electron-emitting radionuclides in tumor imaging and therapy has presented some new challenges to conventional radiation dosimetry. The fraction of electron energy absorbed in most source regions has usually been assumed to be unity. In small structures such as localized tumors or isolated regions containing moderate to high energy electron emitters, however, this may not always be the case. **Methods:** Using an extension of Berger's scaled absorbed dose distributions for point sources to represent a spherical geometry, absorbed fractions of electron energy for sources uniformly distributed in spheres of various sizes have been calculated. **Results:** Beta particle and monoenergetic electron energies studied range from 0.025 to 4.0 MeV and sphere masses range from 0.01 to 1000 g. S values have also been calculated for ^{90}Y, ^{123}I and ^{131}I based on the results of the absorbed fraction calculations. **Conclusion:** These calculated absorbed fractions are valuable in estimating electron energy loss from small spherical structures and may be useful in estimating the radiation dose to these small volumes.

Key Words: beta particles; electron-emitting radionuclides; radiation dosimetry; ^{123}I; ^{90}Y; ^{131}I

J Nucl Med 1994; 35:152–156

Mean radiation doses and dose equivalents traditionally have been calculated for the whole body and certain organs for safety purposes. The Medical Internal Radiation Dose (MIRD) Committee of the Society of Nuclear Medicine published methods for calculating radiation doses (*1*). Published MIRD dose estimates typically express radiation absorbed doses to the whole body and many of the important organs and organ systems of the body.

The mean dose to an organ is usually considered to be a useful indicator of the risk of acute or delayed effects in that organ if (1) the activity is approximately uniformly distributed throughout the organ and (2) the range or mean free path of the principal radiations emitted by the radionuclide is large enough that the absorbed dose throughout the organ is fairly uniform. In the case of electrons, the range also must not be so large relative to the organ dimensions that an appreciable loss of energy occurs due to escape beyond the organ boundaries. For example, ^{131}I (0.183 MeV mean beta energy) NaI is taken up more or less uniformly by the thyroid (19.6 g), and the average dose to the thyroid is the appropriate quantity to calculate in studying radiation effects in the thyroid.

New applications in nuclear medicine, most notably the research into the uses of monoclonal antibodies (Mabs), as well as an increasing general awareness of situations in which the above two assumptions are not satisfied, have led to a need for more detailed dosimetry methods and models.

In the traditional models, often called macrodosimetric methods, the absorbed fraction (fraction of energy emitted in a source region which is absorbed in some target regions) for electrons is assumed to be unity for the source organ and zero elsewhere, i.e., all of the electron energy emitted in a source region is absorbed in that region. This is quite reasonable for most situations because the range of most electrons in body tissues is small compared to the size of most source regions, even relatively small organs like the thyroid.

The use of Mabs has caused some investigators to reexamine the appropriateness of the macrodosimetric model assumptions. Mabs have been shown to have a nonuniform distribution within some of the regions where it is important to estimate radiation dose (*2*), and where they may concentrate moderate to high electron energy emitters in small source regions such as small tumors. Specifically, calculation of the radiation dose to tumors may require consideration of the losses of electron energy. Some tumors are small compared to the range of the emissions from many of the radionuclides for therapeutic applications.

This work provides absorbed fractions for beta particles and monoenergetic electrons uniformly distributed throughout spheres of various sizes, as a function of energy. These absorbed fractions may be of use in estimating the radiation dose to small structures containing electron-emitting radionuclides, such as tumors or small organs (e.g., the fetal thyroid).

Received Apr. 9, 1993; revision accepted Aug. 25, 1993.

For correspondence or reprints contact: Jeffry A. Siegel, PhD, Cooper Hospital, University Medical Center, Dept. of Radiation Oncology, 1 Cooper Plaza, Camden, NJ 08103.

MATERIALS AND METHODS

Berger provided scaled absorbed dose distributions for point sources in an aqueous medium (*3*), which were functions of distance from the source, the x_{90} distance (distance in which 90% of the energy emitted by a point source will be absorbed), and the specific absorbed fraction (absorbed fraction per unit mass at the distance of interest). These point source functions may be integrated to estimate the absorbed dose near extended sources of activity (line, plane, volume). In a related publication (*4*), Berger provided solutions to such extensions of point source functions to some standard geometries, including spheres. The solution for a sphere was employed here for sphere masses of 0.01, 0.1, 0.5, 1, 2, 4, 6, 8, 10, 20, 40, 60, 80, 100, 300, 400, 500, 600 and 1000 g. Solutions of the scaled absorbed dose distributions for average beta particle energies of 0.062, 0.118, 0.183, 0.385, 0.695, 0.937 and 1.428 MeV and monoenergetic electron energies of 0.025, 0.05, 0.1, 0.2, 0.4, 0.7, 1.0, 2.0, and 4.0 MeV were used (*3*). The following formula for an extended source was used:

$$\phi = 4\pi\rho \int_0^\infty x^2\psi(x)\Phi(x)\,dx,$$

where ϕ = the absorbed fraction of energy in the source region; ρ = the medium density; x = the distance from a point source; $\psi(x)$ = the geometric reduction factor for the given geometry at distance x; and $\Phi(x)$ = the specific absorbed fraction of energy at distance x.

The integration is carried out over all x from zero to infinity. The specific absorbed fraction is derived from the scaled absorbed dose distributions for point sources defined in *MIRD Pamphlet No. 7* (*3*):

$$\Phi(x) = \frac{F(x, x_{90})}{4\pi\rho x^2 x_{90}},$$

where $F(x, x_{90})$ = the scaled absorbed dose distribution.

The geometrical reduction factor for a spherical geometry is defined by Berger (*4*) as:

$$\psi(x) = 1 - 1.5(x/d) + 0.5(x/d)^3 \qquad 0 \le x \le d$$

$$\psi(x) = 0 \qquad d < x,$$

where d = sphere diameter.

In these calculations, simple trapezoidal integration was employed starting at x = 0 and using small step sizes out to the limits of the scaled absorbed dose distributions. The values of $F(x, x_{90})$ from Berger's tables were chosen by linear interpolation. Values of x_{90} were also taken from Berger's document for a given beta-particle or electron energy. The resulting absorbed fraction calculations are for spheres of unit density within a scattering medium of unit density.

S-values (*5*) were calculated for ^{90}Y, ^{123}I and ^{131}I for the various sphere sizes considered, using the results of the absorbed fraction calculations. These radionuclides were chosen in order to show how the calculations affect a variety of emitter types; ^{90}Y is a high-energy beta emitter, ^{131}I is a moderate-energy beta emitter and ^{123}I emits mostly low energy conversion electrons. In order to calculate a total S-value for ^{123}I and ^{131}I, absorbed fractions for the photon emissions were also needed (bremsstrahlung from ^{90}Y was neglected). Neglecting bremsstrahlung production results in <1% error in the calculations since <1% of the total beta energy up to 4.0 MeV is converted to photons (*6*). The photon-absorbed fractions were taken from *MIRD Pamphlets* 2 and 8 (*7,8*) for photon emitters uniformly distributed in various spheres. However, absorbed fractions could not be located for spheres of masses less than 1 g, so in these cases the S-values are given for the electron contributions only. The error associated with not including the photon contribution in these S-values is on the order of 1%.

RESULTS

The calculated absorbed fractions for the various sphere sizes as a function of energy are shown in Tables 1 and 2, for beta particles and monoenergetic electrons, and graphically in Figures 1 and 2. The S-values for the three radionuclides studied are given in Table 3.

DISCUSSION

Figures 1 and 2 effectively show the situations in which the modification of the standard formula for absorbed dose in a sphere should include a correction for losses of electron energy. For beta-particle emitters, masses under 10 g and average beta particle energy above about 0.5 MeV, the absorbed fraction drops below 0.9, indicating the error will be no greater than 10% without the correction for larger spheres or lower energies. For monoenergetic electrons, the same argument applies for spheres less than 10 g and electron energies above 0.75 MeV, or perhaps larger spheres at energies above 2 MeV. Because of the small variations in the absorbed fraction over the matrix of energies and sphere sizes, linear interpolation in these tables will yield reasonable values of absorbed fractions for intermediate sizes and energies. Interpolation in Table 1 ignores the differences in beta spectral shapes between the nuclides. Although accounting for the different shapes of allowed beta particle spectra in this analysis is possible (*9*), it would be very hard to generalize for the user as corrections are a function of maximum particle energy and element atomic number. While it is realized that this effect may influence the values in Table 1, the error it introduces into the calculations is minor.

Table 3 shows the importance of this electron energy loss correction for three radionuclides with different emission spectra. For ^{90}Y, a fairly high-energy beta emitter, the effect of the correction is seen to be important for masses under 40 g. For ^{123}I, which has a few moderate to low energy conversion and Auger electrons and several photon emissions, the effect is small at almost all the masses studied. For ^{131}I, which has some medium-energy beta-particle emissions and a few low-energy conversion electrons with many important photon emissions, the effect of the correction is noticeable at masses under 1 g, but is not as significant as for ^{90}Y.

CONCLUSION

Berger's method for estimating the absorbed fraction of electron energy loss from spheres was useful in studying the amount of energy loss in spheres from 0.01 g to 1000 g and beta particle and electron energies up to

TABLE 1
Absorbed Fractions for Beta-Particle Sources Uniformly Distributed in Spheres of 1 g/cm^3 Density

Sphere mass (g)	Sphere radius (cm)	ϕ Average beta energy (MeV)*						
		0.062	0.118	0.183	0.385	0.695	0.937	1.428
0.01	0.13	0.96	0.86	0.77	0.57	0.32	0.23	0.15
0.1	0.29	0.98	0.94	0.89	0.78	0.58	0.44	0.31
0.5	0.49	0.99	0.96	0.94	0.87	0.74	0.63	0.48
1	0.62	0.99	0.97	0.95	0.90	0.79	0.70	0.56
2	0.78	1.00	0.98	0.96	0.92	0.83	0.76	0.64
4	0.98	1.00	0.98	0.97	0.94	0.87	0.81	0.71
6	1.13	1.00	0.99	0.98	0.95	0.88	0.83	0.74
8	1.24	1.00	0.99	0.98	0.95	0.90	0.85	0.76
10	1.34	1.00	0.99	0.98	0.96	0.90	0.86	0.78
20	1.68	1.00	0.99	0.98	0.97	0.92	0.89	0.82
40	2.12	1.00	0.99	0.99	0.97	0.94	0.91	0.86
60	2.43	1.00	1.00	0.99	0.98	0.95	0.92	0.88
80	2.67	1.00	1.00	0.99	0.98	0.95	0.93	0.89
100	2.88	1.00	1.00	0.99	0.98	0.96	0.93	0.90
300	4.15	1.00	1.00	1.00	0.99	0.97	0.96	0.93
400	4.57	1.00	1.00	1.00	0.99	0.97	0.96	0.94
500	4.92	1.00	1.00	1.00	0.99	0.98	0.96	0.94
600	5.23	1.00	1.00	1.00	0.99	0.98	0.96	0.94
1000	6.20	1.00	1.00	1.00	0.99	0.98	0.97	0.95

*The average beta energies in this table correspond to the following radionuclides: 0.062 = ^{147}Pm; 0.118 MeV = ^{59}Fe; 0.183 MeV = ^{131}I; 0.385 MeV = ^{11}C; 0.695 MeV = ^{32}P; 0.937 MeV = ^{90}Y; and 1.428 MeV = ^{106}Rh.

4.0 MeV. The correction for loss of electron energy is important in spheres smaller than 10 g and for electron energies above 0.5 MeV. This correction can be important for radionuclides which emit primarily moderate to high energy beta particles, but is less important for nuclides with lower-energy beta particles or electrons or for which photon emissions dominate the decay schemes. The tables presented here can be applied to many situations involving beta particles or electron emitters in spherical objects.

TABLE 2
Absorbed Fractions for Monoenergetic Electron Sources Uniformly Distributed in Spheres of 1 g/cm^3 Density

Sphere mass (g)	Sphere radius (cm)	ϕ Electron energy (MeV)								
		0.025	0.05	0.1	0.2	0.4	0.7	1.0	2.0	4.0
0.01	0.13	1.00	1.00	0.97	0.89	0.67	0.36	0.20	0.093	0.047
0.1	0.29	1.00	1.00	0.99	0.96	0.85	0.67	0.50	0.21	0.10
0.5	0.49	1.00	1.00	1.00	0.98	0.92	0.80	0.70	0.39	0.18
1	0.62	1.00	1.00	1.00	0.99	0.94	0.85	0.76	0.49	0.23
2	0.78	1.00	1.00	1.00	0.99	0.95	0.88	0.81	0.58	0.29
4	0.98	1.00	1.00	1.00	1.00	0.96	0.91	0.85	0.66	0.38
6	1.13	1.00	1.00	1.00	1.00	0.97	0.92	0.87	0.70	0.44
8	1.24	1.00	1.00	1.00	1.00	0.97	0.93	0.88	0.73	0.48
10	1.34	1.00	1.00	1.00	1.00	0.98	0.93	0.89	0.75	0.51
20	1.68	1.00	1.00	1.00	1.00	0.98	0.95	0.91	0.80	0.59
40	2.12	1.00	1.00	1.00	1.00	0.99	0.96	0.93	0.84	0.67
60	2.43	1.00	1.00	1.00	1.00	0.99	0.97	0.94	0.86	0.71
80	2.67	1.00	1.00	1.00	1.00	0.99	0.97	0.95	0.87	0.73
100	2.88	1.00	1.00	1.00	1.00	1.00	0.97	0.95	0.88	0.75
300	4.15	1.00	1.00	1.00	1.00	1.00	0.98	0.97	0.91	0.82
400	4.57	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.93	0.84
500	4.92	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.93	0.85
600	5.23	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.94	0.86
1000	6.20	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.95	0.88

TABLE 3
S-Values for ^{90}Y, ^{123}I and ^{131}I Uniformly Distributed in Spheres of 1 g/cm³ Density

	S-Value (rad/μCi-hr)					
	^{90}Y		^{123}I		^{131}I	
Sphere mass (g)	Uncorrected	Corrected	Uncorrected	Corrected	Uncorrected	Corrected
0.01	200	46	6.0*	5.8*	40*	31*
0.1	20	8.8	0.60*	0.59*	4.0*	3.6*
0.5	4.0	2.5	0.12*	0.12*	0.81*	0.76*
1	2.0	1.4	0.066	0.066	0.41	0.39
2	1.0	0.76	0.034	0.034	0.21	0.20
4	0.5	0.40	0.017	0.017	0.10	0.10
6	0.33	0.28	0.012	0.012	0.070	0.069
8	0.25	0.21	0.0090	0.0090	0.053	0.052
10	0.20	0.17	0.0073	0.0073	0.043	0.042
20	0.10	0.089	0.0038	0.0038	0.021	0.021
40	0.050	0.045	0.0020	0.0020	0.011	0.011
60	0.033	0.031	0.0015	0.0015	0.0075	0.0074
80	0.025	0.023	0.0012	0.0012	0.0057	0.0057
100	0.020	0.018	0.00097	0.00097	0.0046	0.0046
300	0.0066	0.0064	0.00037	0.00037	0.0016	0.0016
400	0.0050	0.0048	0.00029	0.00029	0.0012	0.0012
500	0.0040	0.0038	0.00024	0.00024	0.0010	0.0010
600	0.0033	0.0032	0.00020	0.00020	0.00084	0.00084
1000	0.0020	0.0019	0.00013	0.00013	0.00052	0.00052

*Beta and electron contributions only.

ACKNOWLEDGMENTS

This work was performed for the U.S. DOE under contract DE-AC05-76OR00033 and for the U.S. FDA under Interagency Agreement No. FDA 224-75-3016, DOE 40-286-71. The submitted manuscript has been authored by a contractor of the U.S. Government under contract DE-AC05-76OR00033. Accordingly, the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of the contribution, or allow others to do so, for U.S. Government purposes.

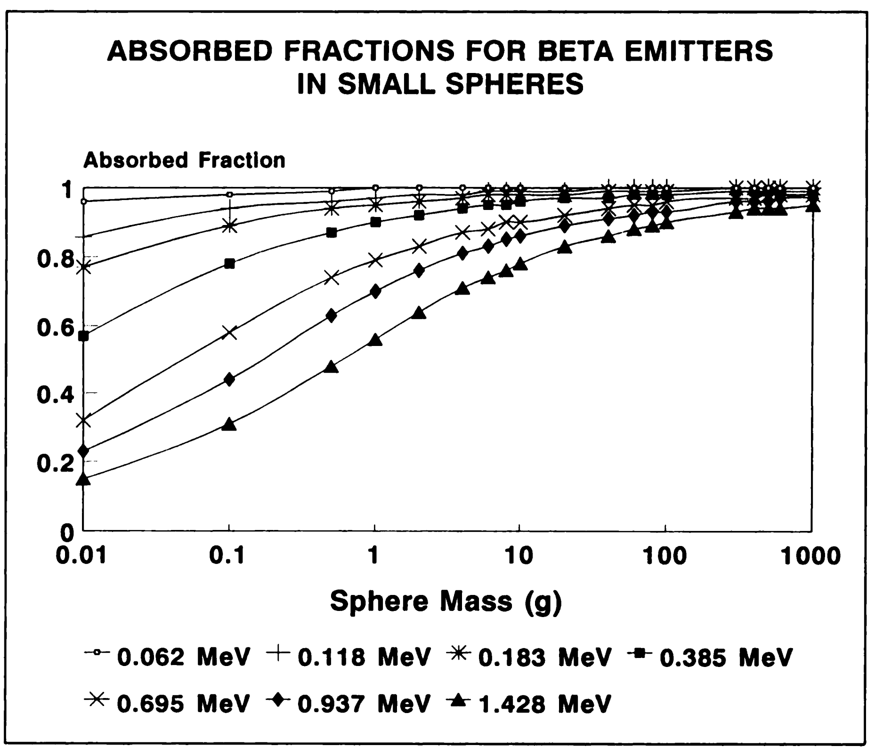

FIGURE 1. Plot of the absorbed fraction for beta sources in spheres of density 1 g/cm³ as a function of sphere mass and energy.

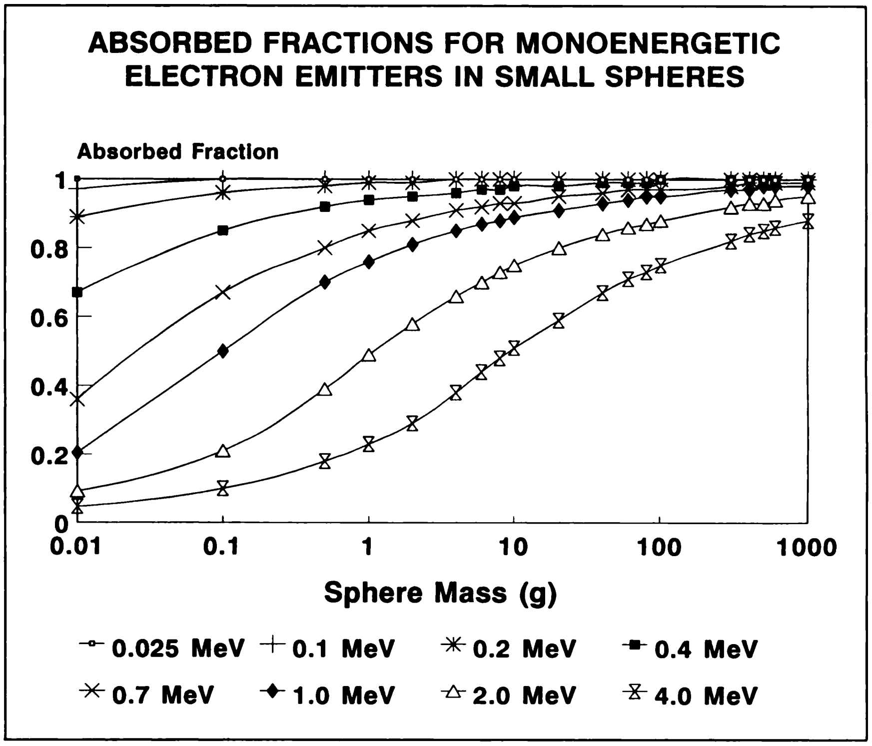

FIGURE 2. Plot of the absorbed fraction for monoenergetic electron sources in spheres of density 1 g/cm^3 as a function of sphere mass and energy.

REFERENCES

1. Loevinger R, Berman M. A revised schema for calculating the absorbed dose from biologically distributed radionuclides. *MIRD pamphlet No. 1*, revised. New York: Society of Nuclear Medicine, 1976.
2. Howell R, Rao D, Sastry K. Macroscopic dosimetry for radioimmunotherapy: nonuniform activity distributions in solid tumors. *Med Phys* 1989;16: 66–74.
3. Berger M. Distribution of absorbed dose around point sources of electrons and beta particles in water and other media. MIRD Pamphlet No. 7. *J Nucl Med* 1971;12(suppl 5):5–23.
4. Berger M. Beta-ray dosimetry calculations with the use of point kernels. In: *Medical radionuclides: radiation dose and effects*. Oak Ridge: U.S. Atomic Energy Commission; 1970:63–86.
5. Snyder W, Ford M, Warner G, Watson S. "S," absorbed dose per unit cumulated activity for selected radionuclides and organs. *MIRD pamphlet no. 11*. New York: Society of Nuclear Medicine; 1975.
6. Cember H. *Introduction to health physics*. New York: Pergamon Press; 1969:118.
7. Berger M. Energy deposition in water by photons from point isotropic sources. MIRD pamphlet no. 2. *J Nucl Med* 1968;9(suppl 1):15–25.
8. Ellett W, Humes R. Absorbed fractions for small volumes containing photon-emitting radioactivity. MIRD Pamphlet No. 8. *J Nucl Med* 1971; 12(suppl no. 5):25–32.
9. Loevinger R, Japha E, Brownell G. Discrete radioisotope sources. In: Hine G, Brownell G, eds. *Radiation dosimetry*. New York: Academic Press; 1956:693–799.