Fast Maximum-Likelihood Reconstruction

Tom R. Miller and Jerold W. Wallis

The Edward Mallinckrodt Institute of Radiology, Washington University School of Medicine, St. Louis, Missouri

A significant problem in application of maximum-likelihood reconstruction to SPECT and PET studies is the very long computation time required for this iterative algorithm. An approach is presented in which the basic calculations involved in projection and backprojection are performed only once and stored in computer memory, thus significantly reducing the computation time.

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The very long computation time of the iterative maximum-likelihood algorithm hampers the application of this reconstruction method in single-photon tomography and positron tomography. Here we describe an approach that yields maximum-likelihood reconstructions with conventional computers in times that are modest for research purposes and that approach clinically acceptable values. The basic idea, mentioned by several previous authors [e.g., (*1,2*)], involves trading memory for speed. The computationally burdensome projection, backprojection, attenuation and blurring calculations are performed only once with the weighting coefficients stored in memory. Then, for each iteration those coefficients are recalled from memory to yield the iterated images quickly.

METHODS

Maximum-likelihood reconstruction is usually performed with the iterative expectation-maximization algorithm relating the estimate of the radioactivity distribution at the $(k + 1)^{th}$ iterate $\lambda^{(k+1)}(x)$ to the value $\lambda^{(k)}(x)$ at the kth iteration for all positions x in the reconstructed slice. Assuming that $M_\theta(\mathbf{du})$ is the number of counts detected in the interval $\mathbf{du}$ for N_θ angles, the equation for the expectation-maximization algorithm (*3*) is

$$\lambda^{(k+1)}(x) = \lambda^{(k)}(x)\left(\frac{1}{\bar{\beta}(x)N_\theta}\sum_\theta \int \frac{q(\mathbf{u}\,|\,x,\theta)M_\theta(\mathbf{du})}{\int q(\mathbf{u}\,|\,z,\theta)\lambda^{(k)}(z)dz}\right).$$

The superposition matrix $q(\mathbf{u}\,|\,x,\theta)$ is the probability that a photon emitted at position x is detected at $\mathbf{u}$. q is the product of the gamma-ray attenuation and the point spread function (PSF). The attenuation term $\bar{\beta}$ is the probability that a photon will be detected at any angle and projection position.

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For reprints contact: Tom R. Miller, MD, PhD, The Edward Mallinckrodt Institute of Radiology, 510 S. Kingshighway Blvd., St. Louis, MO 63110.

To achieve rapid iterations, the discrete form of the q-matrix is computed once and stored in computer memory. Then, at each iteration, the values of q are read from the memory and the floating-point calculations are performed, thus eliminating the need to recompute the attenuation and blurring at each iteration. The q-matrix consists of 25 million elements for a 64 × 64 pixel transaxial slice with 96 projection angles with 64 pixels at each angle. Thus, 100 megabytes of memory would be required if the values are stored as four-byte floating-point words. This large number can be substantially reduced by use of scaled two-byte integer storage and reduction in the number of projection points at each angle. The number of projection points can be further reduced by a factor of four by not storing q-matrix values very close to zero, e.g., by storing only 17 projection points for each x and θ, giving 8 pixels on each side of the ideal, unblurred projection point. Thus, the q-matrix can be stored in approximately 13.3 megabytes of memory ((100/2) × (17/64)) without loss of accuracy. If data are collected at only 64 projection angles, then the storage requirement would be 8.9 megabytes. An additional reduction in memory of approximately 21% can be achieved if reconstruction is performed only for a circular region inscribed in the square transaxial slice. Thus, only 7–10 megabytes of memory are required for 64–96 projection angles.

An inspection of the equation shows that there are approximately 27 million floating-point operations per iteration for a 64 × 64 pixel matrix with 96 projection angles when the PSF is limited to 17 pixels; each element in the summation requires both a multiplication and an addition operation (64 × 64 × 17 × 96 × 2 × 2). If reconstruction is restricted to the inscribed circle, then there are 21 million floating-point operations.

The calculations described above were implemented on a modern reduced-instruction-set computer (RISC) machine, the DECstation 5000 (Digital Equipment Corp., Maynard, MA) with 32 megabytes of memory. The speed of the DECstation is 24 million instructions per second (MIPS).

RESULTS

Computation times for the calculation of the q-matrix and for the performance of 50 or 100 iterations of the expectation-maximization algorithm are shown in Table 1 for typical data acquisition parameters and numbers of iterations. Results are presented for the conventional method, where the q-matrix is calculated at each iteration, and for the new method involving only one calculation of q. A simple pixel-driven approach was used to compute the attenuation matrix. More elaborate calculations would lengthen computation of the q-matrix but would have no effect on the time per iteration and, hence, little effect on total computation time.

TABLE 1
Measured Computation Times for the Conventional and the New Maximum-Likelihood Algorithms

Acquisition and Iterations	q-matrix setup (sec)	1 iteration (sec)	Conventional method* (min)	Pre-calculated q-matrix† (min)
64 × 64 matrix 96 angles 100 iterations	59.4	7.5	111.5	13.5
64 × 64 matrix 64 angles 50 iterations	39.6	5.0	37.2	4.8

* Time = no. of iterations × (q-matrix setup + 1 iteration).
† Time = q-matrix setup + no. of iterations × 1 iteration.

DISCUSSION

We have described a simple approach that leads to rapid reconstructions with conventional, widely available computers. While this method has been mentioned in numerous previous publications [e.g. (*1,2*)], it was always dismissed as impractical because of a lack of sufficient computer memory. The recent conjunction of two factors has made this computational technique practical. First, the required large quantities of memory can now be purchased at low cost. Perhaps of equal importance is the speed of modern RISC computers, typically 10–100 times faster than popular minicomputers, such as the microVAX II, thus allowing the iterations to proceed quickly.

The computation speeds achieved here will likely be superseded in the near future by parallel implementation of the algorithm, or by optimized estimation methods (*2*), or well-chosen initial estimates that may reduce the required number of iterations below the 50–100 iterations used in Table 1. State-of-the-art hardware, with two processors running in parallel, should be able to perform 50 iterations at 64 angles in less than 1 min for a single slice. Thus, the central 30–40 slices of a 64-slice clinical SPECT study could be reconstructed in 30 min and, if an optimized searching algorithm is developed, essentially all slices of a study could be reconstructed in that time.

The maximum-likelihood approach is formulated here for reconstruction of one or more two-dimensional transaxial slices employing, for SPECT, a parallel-hole collimator. The use of fan-beam collimation would lengthen the projection and backprojection operations slightly during generation of the q-matrix but would not significantly increase the size of the matrix or the computation time of each iteration. A true three-dimensional reconstruction requires consideration of the interaction between slices. Such an algorithm would require a two-dimensional PSF, thus significantly lengthening the computation time. The use of cone-beam or astigmatic collimation also requires a three-dimensional implementation.

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