
A Theoretical Comparison of First-Pass and Gated Equilibrium Methods in the Measurement of Systolic Left Ventricular Function

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First-pass and gated equilibrium radionuclide studies of left ventricular function have proven extremely useful in the detection and management of patients with heart disease. Despite this practical experience, however, comparison of these methods generally has been confined to procedural differences that do not reflect the intrinsic properties of the methods. Here, we describe the results of a simple theoretical calculation from first principles that compares the methods based on their relative statistical precision. This analysis assumes that each procedure is carried out with the same tracer dose in the same hypothetical patient under identical conditions and with the same ideal imaging equipment. Results obtained with this model suggest that the imaging time required for a gated equilibrium study to achieve the same statistical precision as a first-pass study is typically less than 2 min in resting subjects and less than 1 min during stress. The analysis also indicates that gated equilibrium studies will tend to possess the greater statistical precision when cardiac output is elevated, such as when the heart is imaged during exercise. On the other hand, this analysis indicates that the first-pass method will tend to possess the greater precision when cardiac output is low and when imaging time is highly constrained.

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First-pass and gated equilibrium radionuclide methods for measuring systolic left ventricular (LV) function (1-17) have been in widespread use for many years. These two methods have most often been compared in terms of procedural differences such as the camera orientation usually used to view the heart, technical factors related to ease of use, differences in tracer requirements for the two methods, and so on. While both methods are thought to produce valid estimates of the same systolic LV function parameters, the methods often differ in the statistical precision of these measurements. Therefore, a fundamental

comparison of the methods can be made by asking which method provides the most statistically reliable measurement of some given systolic LV function parameter. Here "statistical reliability" is measured by the uncertainty in the chosen parameter due solely to the cumulative effect of counting uncertainties in the (random) variables needed to compute that parameter. In the case of ejection fraction, for example, counting uncertainties in three variables contribute to this error: the counting fluctuations in gross LV end-diastolic, gross end-systolic, and background counts. Such statistical errors, of course, represent only one of several significant sources of error in real studies. This particular error, however, is irreducible in the sense that even if all other sources of error in the determination of some parameter could be eliminated and all assumptions regarding background correction, region of interest (ROI) definition, etc., are true, this source of error will remain. Precision thus measures a fundamental comparative property of the methods, one that will have meaning even when both methods are perfectly implemented (as we shall assume).

Given this basis for comparison, we can further imagine performing both studies on the same hypothetical patient using the same tracer dose, same perfect camera, same collimator and same camera projection to insure identical detection efficiency for photons emitted from the ventricle. We might first perform the first-pass study on this subject, compute the parameter of interest, and then compute the uncertainty in this parameter due to counting fluctuations in the component variables. We could then perform a gated equilibrium study on this same subject, compute the same parameter (which should have a value similar in magnitude to that obtained with the first-pass method) and then compute the error in this estimate of the parameter. If all other factors could be ignored, the method possessing the greater statistical precision (smallest uncertainty due to counting fluctuations) would be the method of choice for that particular measurement circumstance. We can put these ideas into practice by recognizing that the duration of a gated equilibrium study is, at least in principle, a "free" variable so that there will always exist some gated equilibrium imaging time (the "equivalence

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time”) that will make the error in the chosen parameter the same as the error computed for the first-pass study. In this work, we chose to use the background-corrected value of LV end-diastolic counts as the parameter for comparison and the uncertainty in this parameter due to counting fluctuations as the requisite error. The argument presented here does not depend strongly on the choice of parameter and we have chosen net end-diastolic counts primarily to simplify the calculation. Thus, we derive an expression for the gated equilibrium equivalence time required for a gated equilibrium study to attain the same precision of measurement of net LV end-diastolic counts as a first-pass study performed under exactly the same circumstances. This equivalence time is found to depend on the functional state of the heart, so we evaluate this result for two common clinical applications, measurement of LV systolic function in adult subjects at rest and during exercise.

METHODS

Unless otherwise noted, all variables measuring LV counts are assumed to be corrected for background and are thus “net” counts. A variable not corrected for background will be described as measuring “gross” counts.

First-Pass Method

If a tracer bolus is rapidly introduced into the venous circulation and observed during its initial passage through the left ventricle, it can be shown (Appendix) that the cumulative amount of radioactivity present in the LV at end-diastole is given by the expression Q/EF , where Q is the total injected activity and EF is the total ejection fraction (all symbols used in the text, and their units of measurement, are defined in the Appendix). This result is predicated on a single, complete passage of the bolus through the central circulation without venous return, patent valves, perfect and instantaneous mixing of incoming activity with LV blood and summation of all end-diastolic activities from the moment tracer enters the LV until no tracer remains in the LV, i.e., this sum includes the “wash-in” phase as well as the entire wash-out phase of the bolus transit. We also assume that LV function is stable during the bolus passage. Use of cumulative activity is the basis for the “representative cycle” method of creating a cardiac image sequence from first-pass data (7). This, and similar methods, maximize use of the available first-pass data by superimposing image data acquired during consecutive cardiac cycles to yield a single, average cycle with enhanced signal-to-noise properties.

If this cumulative LV end-diastolic activity is now sampled with an imaging device of efficiency, e , for LV activity for a period equal to the duration of the end diastolic image, t , the total net number of counts from the LV in this first-pass end-diastolic image, NF , will be: $NF = (Q)(t)(e)/EF$. As expected, NF increases with tracer dose, duration of the end-diastolic image, and with detection efficiency of the imaging device for LV activity. Perhaps less obvious, NF also is modulated by the EF , decreasing as EF increases. It follows from this calculation that the net counts accumulated from the LV depend on the functioning of the LV. NF will be relatively large in patients with poor LV function, i.e., low EF , and relatively smaller in patients with good or enhanced LV function, i.e., normal to high EF s. Examples from both extremes might include patients with severe,

prior myocardial infarction (low EF s) and some patients with aortic stenosis or hypertrophic cardiomyopathy (high EF s).

Gated Equilibrium Method

A dose, Q , of a blood-labeling radiotracer is administered to a patient with total blood volume, V , and LV end-diastolic volume, V_{ed} . After tracer equilibration, the patient is imaged for M cardiac cycles during which LV function is stable. The total net number of LV counts accumulated in the end diastolic image of this gated equilibrium study, NE , will be: $NE = (Q)(t)(e)(V_{ed})(M)/V$. Total net end-diastolic LV counts in a gated equilibrium study increases with tracer dose, duration of the end diastolic image, detection efficiency, the fraction of the total blood volume that resides in the LV, and with the number of cardiac cycles sampled. Unlike the first-pass method, NE does not depend directly on LV function, but rather on relative LV size. If V and all other factors remain constant, NE will be larger in patients with dilated LVs and smaller in patients with small LVs. Examples from both extremes might include patients with severe, chronic aortic regurgitation (large LVs) and patients with hypertrophic cardiomyopathy (small LVs).

Comparison of First-Pass and Gated Equilibrium Methods

If it is assumed that both procedures are carried out in the same hypothetical patient using the same “perfect” imaging device, tracer dose, image duration and the same detection efficiency for LV activity, then the ratio of net end diastolic counts for the two methods is:

$$NE/NF = (EF)(V_{ed})(HR)(T)/V,$$

where the number of heart beats, M , has been replaced by the product of heart rate (HR) and total gated equilibrium imaging time (T). This expression can be further simplified since the product $(EF)(V_{ed})(HR) = CO$ (total cardiac output). Thus, when both methods are applied in the same patient, and when dose, image duration, and detection efficiency are the same, the ratio of net end-diastolic LV counts for the two methods becomes:

$$NE/NF = (CO)(T)/V. \quad \text{Eq. 1}$$

According to this expression, the relative number of LV photons acquired by the two methods is proportional to total cardiac output and the time available for gated equilibrium imaging, and inversely proportional to total blood volume. The ratio, CO/V , is also the inverse of the mean transit time (MTT) for a particle moving through a volume V at flow rate CO . Therefore, the ratio NE/NF also could be written as $NE/NF = T/MTT$, the ratio of gated equilibrium imaging time to MTT through the blood pool. The form of this relationship indicates that MTT can be regarded as the effective duration of a first pass study, just as T is the actual duration of a gated equilibrium study.

Modifications to Equation 1

We derived Equation 1 by assuming that both methods were implemented in their ideal forms. In practice, however, neither method is “ideal” and Equation 1 should be modified to more closely reflect reality. In addition, a significant intermethod difference exists in “background fraction” that also requires modification of Equation 1.

In deriving NF , we assumed that the entire bolus transit contributes to the representative cycle when, in practice, only a fraction of the transit can be incorporated. This fraction is difficult to estimate since it depends on factors such as the arrival time of

activity making a second-pass through the left ventricle and interrupting the washout of the initial bolus transit. A plausible, maximum value for this fraction, however, is three-fourths, and we shall use this value to modify Equation 1. In the derivation of NE, we assumed that all injected tracer remains in the blood pool. However, some activity is excreted and thus is unavailable for imaging. The fraction of activity present in the blood pool depends on labeling method, on elapsed time since injection, and on the isotopic decay rate. For ^{99}Tc -labeled red blood cells and plausible imaging times, this factor is about three-fourths, a reasonable compromise across existing labeling methods and imaging procedures (17).

A more subtle, practical difference between these methods requires correction. NE and NF are assumed to be net end-diastolic LV counts. In practice, both NE and NF are computed as the difference between gross end-diastolic counts and "background" counts, counts detected over the LV but emanating from non-LV structures. The representative cycle first-pass method has the lower background fraction (about 35% of gross end-diastolic counts versus 50%) (9,17). This difference means that a larger number of gross LV counts must be accumulated in a gated equilibrium study to achieve the same relative uncertainty in net end-diastolic counts as can be achieved at lower gross end-diastolic counts in a first-pass study. The magnitude of this effect can be estimated by calculating the ratio of relative standard deviations of net LV end-diastolic counts for the two methods. If $f = (s_F/s_E)$, where s_F and s_E are the relative standard deviations of net LV end-diastolic counts for the first-pass and gated equilibrium methods, respectively, f is found to be related to the ratio of net LV end-diastolic counts by the (approximate) expression: $NE/NF = 3/2 (f^2)$. In deriving this relationship, we have assumed the background fractions for the methods noted previously and have also assumed that the relative sizes of background and LV ROIs are similar for the two methods. When f , a direct measure of the relative precision of the methods, takes on the value $f = 1$, NE and NF have equal statistical precision and $NE/NF = 3/2$. Thus, net gated equilibrium end-diastolic LV counts must be about 50% larger than net first-pass end-diastolic LV counts in order to achieve equal precision. Equivalently, gated equilibrium imaging time must be extended by a factor of 3/2 to overcome the difference in background fractions. If this expression for NE/NF is inserted into Equation 1, the other correction factors included, and the resulting expression rearranged, the result is:

$$T = (3/2)(V/CO)(f^2), \quad \text{Eq. 2}$$

the gated equilibrium imaging time required for a gated equilibrium study to achieve a relative precision, f , in net end-diastolic LV counts, compared to a first-pass study performed in this same subject under identical circumstances.

Applications

The condition $f = 1$ in Equation 2 defines a functional boundary between the first-pass and gated equilibrium methods. In a patient with given blood volume and cardiac output, gated equilibrium imaging for a period $T = (3/2)(V/CO)$ will yield a gated equilibrium study that is statistically identical to a first-pass study performed in this same patient (using, as we have assumed, the same tracer dose, image duration, etc.). In this case, the methods cannot be distinguished intrinsically from one another and so can be considered "equivalent." It may happen, however, that circumstances permit the actual gated equilibrium imaging time to exceed the equivalence time in that particular application. If

so, the precision of measurement of the gated equilibrium study will exceed that of the first-pass study. Conversely, if the time available for gated equilibrium imaging is less than the equivalence time for that application, the precision of the first-pass study will exceed that of the gated equilibrium study. It is clear that in general the curve $T = (3/2)(V/CO)$ will divide the (T, CO) plane into two regions, one where the gated equilibrium method has the smaller statistical error, and another where the first-pass method has the smaller error. Given a value of V , these two regions can be visualized by plotting equation 2 for $f = 1$. Such a plot is shown in Figure 1, where we have assumed a typical total blood volume of 5 liters (18). The region above and to the right of the curve labeled $f = 1$ is the region where the gated equilibrium method has the greater statistical precision, while the region below and to the left of this curve is the region where the first-pass method has the greater statistical precision. We use this graph to compare semi-quantitatively the relative performance of the two methods in two common clinical applications, the assessment of LV systolic function in adult subjects at rest and during stress.

Measurements at Rest

Perhaps the most common clinical use of either method is to measure systolic LV function in resting subjects. At rest, cardiac output averages about 6 liters/min in normal subjects and about 5.5 liters/min in patients exhibiting moderate symptoms of common heart diseases such as coronary artery disease (8). If a convenient value of 5 liters/min is assumed, Figure 1 shows that at this CO the gated equilibrium imaging time for equivalence ($f = 1$) with the first-pass method is 1.5 min. In most laboratories, gated equilibrium imaging is usually performed in resting subjects for periods greater than 1.5 min. Taking 5 min as a more typical value, Figure 1 shows that the point $T = 5$ min, $CO = 5$ liters/min, lies well above the curve $f = 1$ at about $f = 1.8$. Thus, a gated equilibrium imaging study performed in a typical resting adult would possess a relative uncertainty in net LV end-diastolic counts some 1.8 times smaller than the uncertainty in net LV end-diastolic counts recorded in a first pass study performed in this same individual.

Measurements During Stress

First-pass and gated equilibrium methods are also frequently used to measure systolic left ventricular function during stress. Such "stress testing" permits assessment of functional reserve, enhances detection of underlying heart disease, and can often aid in prognosis and patient management (8-16). During stress, cardiac output can increase by factors ranging from about 2 in patients with moderate heart disease to 3 or more in normal subjects (8). If we assume a factor of 2 increase with stress, from 5 liters/min to 10 liters/min, Figure 1 indicates a corresponding gated equilibrium equivalence time of 45 sec. Assuming 2 min as a more typical value for the duration of gated equilibrium imaging during stress (14), the point $T = 2$ min, $CO = 10$ liters/min lies above the equivalence curve, $f = 1$, at about $f = 1.6$. The uncertainty in net LV end-diastolic counts for a gated equilibrium study performed under these conditions would thus be 1.6 times smaller than for the corresponding first-pass study.

DISCUSSION AND CONCLUSION

The purpose of the present work was to compare objectively first-pass and gated equilibrium methods when both are applied to the same clinical measurement problem. By removing differences in all external factors, applying the

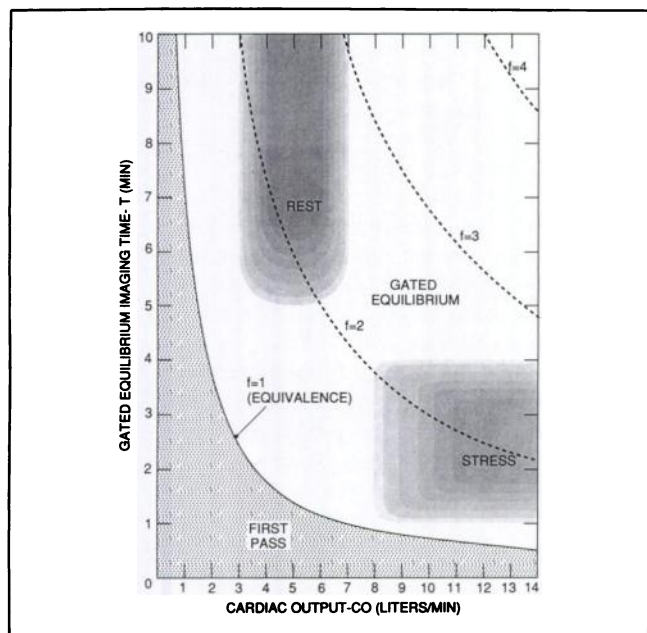


FIGURE 1. Gated equilibrium imaging time (T) versus cardiac output (CO) when total blood volume = 5 liters. The "equivalence" curve labeled $f = 1$ (equal relative standard deviations in first-pass and gated equilibrium net end-diastolic LV counts) divides the T,CO plane into two regions. For T,CO points above and to the right of this curve, f is greater than unity and the gated equilibrium method will possess the smaller statistical uncertainty. For T,CO pairs below and to the left of this curve (dotted region), f will be less than unity and the first-pass method will possess the smaller statistical uncertainty. Shaded regions represent typical ranges for T,CO values at rest and during stress.

methods to the same idealized patient and using a statistical criterion as the basis for comparison, only intrinsic differences remain. Although this treatment is entirely theoretical, two generalizations appear justified by this analysis. First, the gated equilibrium equivalence time does not differ appreciably in an absolute sense from the time required to perform a real first pass study in typical patients. Figure 1 indicates that equivalent gated equilibrium studies can be obtained in typical resting patients in less than 2 min ($CO > 4$ liters/min) and in patients during stress in less than 1 min ($CO > 8$ liters/min). Thus, equivalence can be obtained between the methods for absolutely short gated equilibrium data acquisitions. The first-pass method has been perceived historically as requiring substantially shorter data collection intervals than the gated equilibrium method. The analysis presented here suggests that this perception is largely illusory in typical applications, and arises primarily because acquisition time, by itself, is an inappropriate comparator. When adjusted in imaging time to yield the same statistical precision as first-pass studies, gated equilibrium studies also required brief absolute periods of data acquisition.

Second, while the gated equilibrium equivalence time appears comparable to the time required to perform typical "real" first-pass studies, inspection of Figure 1 indicates that there are circumstances where the methods may pos-

sess very different statistical precisions. When cardiac output is very low and the time available for gated equilibrium imaging very short, the first-pass method may well yield the smaller statistical uncertainty. Conversely, when cardiac output is normal or elevated and the time available for gated equilibrium imaging largely unrestricted, the gated equilibrium method will likely yield the smaller uncertainty. These observations suggest that Equation 2 may help predict the relative behavior of these methods in extreme cases where cardiac output, total blood volume and the likely period of LV functional stability can be estimated. For example, gated equilibrium imaging in normal subjects during exercise would probably yield much smaller errors since CO would likely be large and maintainable at a constant level for extended periods. Conversely, the first-pass method might yield smaller errors during exercise in the subset of patients with severe heart failure due to systolic dysfunction. Such patients, many with low resting cardiac outputs, often can exercise only briefly and increase CO only slightly (if at all) before symptoms occur and, possibly, the functional state of the LV is altered. In such cases, CO would be low and gated equilibrium imaging time short, so that the first-pass method would likely exhibit the smaller uncertainty.

As we noted in the introduction, our purpose in this work was to use a quantitative and unbiased criterion to compare the methods, a criterion based on equality in some measurable variable, i.e., uncertainty in net LV end-diastolic counts. This choice (or any similar choice) of comparability criterion has certain limitations. In addition, the validity of the simple theoretical argument that we present is dependent on a number of assumptions. A few comments regarding validity and comparability are thus in order.

When "real" first-pass or gated equilibrium studies are performed in patients, many assumptions are automatically (though invisibly) invoked. An examination of the assumptions that we have made in deriving Equation 2 will reveal that most of these assumptions are the same as for real studies. In real studies, for example, it is assumed implicitly that background correction of gross LV counts truly compensates for LV background and that the remaining counts represent those emanating from the LV as if it were in isolation from other structure. Our derivation is thus no more or less accurate because of these kinds of assumptions than are real studies. It is possible, moreover, to introduce corrections into the derivation that can, in a general way, compensate for certain violations. We have already included three approximate "corrections" in the derivation of Equation 2: correction for tracer loss from the blood pool in gated equilibrium studies, for recirculation in first-pass studies, and correction for the inter-method difference in background fractions. These particular corrections were included because they arise from within the subject under study and not from external factors. It is possible, however, to include corrections for

other external factors associated with the imaging process. For example, a factor correcting for data loss at the high data rates associated with the first-pass method could be included in the derivation if such losses were significant. Such modifications, however, are likely to constitute relatively minor adjustments to the expressions for end-diastolic counts that form the starting point for our calculation. The largest departure from Equation 2 that real studies might exhibit is most likely to be due to an effect that we have already attempted to recognize, the assumption that we can observe the first transit without interruption until negligible levels of activity remain in the ventricle. This assumption cannot be rigorously correct since, in principle, an infinite period of time is required to wash all activity from the LV by successive dilutions. In reality, recirculation will occur at some moment during the wash-out phase and the number of end-diastolic counts recorded up to that moment will depend to a greater or lesser degree on bolus shape and on ventricular function (see Appendix). If the first transit is essentially complete when recirculation occurs, Equation 2 will likely give a valid estimate of the equivalence time. On the other hand, if recirculation occurs when LV activity levels are still high, Equation 2 will only approximate the equivalence time. This effect is variable and unpredictable since bolus shape and the factors that govern the time to recirculation in any given subject are unknown. It would be expected that this effect would complicate application (or verification) of Equation 2 in practice.

Finally, we should note that the statistical criterion selected for comparing the methods is not without limitations. Large differences in relative statistical precision might exist between the methods but the absolute precision of measurement of either method might be small compared to other sources of error. In such cases, either method would yield an acceptable result. On the other hand, there are circumstances where relative statistical precision is of direct importance. For example, if the intention of a study is to functionally "map" regional LV wall motion or to measure parameters that are strongly influenced by counting "noise", e.g., peak ejection rate, etc., relative statistical precision would be a useful methodologic discriminator if other factors were of secondary importance.

First-pass and gated equilibrium methods of assessing LV systolic performance are in widespread use because both quickly and safely provide information essential to the diagnostic process. Both methods have certain operational features that make one or the other attractive in certain clinical settings. Indeed, in some settings these external features may be of overriding importance and all other factors incidental. Nonetheless, both methods are subject to uncertainties that arise from the fundamental process of radioactive decay even if both methods are otherwise "perfectly" implemented. Comparing the methods at this level reveals a simple relationship for the

imaging time required for a gated equilibrium study to achieve statistical parity with a first-pass study. This equivalence time, which is of the order of a minute or two in typical subjects, increases with increasing total blood volume and decreases with increasing cardiac output. This relationship, with modifications appropriate to the conditions at hand, may help clarify and refine the application of these methods in practice.

APPENDIX

Derivation of Cumulative First-Pass End-Diastolic Activity

Cumulative left ventricular end diastolic activity for an idealized first-pass study can be calculated in three steps:

1. Calculate the recursion relation connecting end-diastolic activities in consecutive cardiac cycles.
2. Use this relation to calculate the total end-diastolic activity in each cardiac cycle beginning with the first cycle in which activity enters the LV.
3. Add these total end-diastolic activities together to obtain the cumulative end-diastolic activity for the entire bolus transit. We assume initially that there is no valvular regurgitation.

The recursion relation connecting end-diastolic activities in consecutive cardiac cycles can be deduced by recognizing that the amount of activity present at the beginning of a cycle is equal to the sum of two activities: the activity present at end-systole in the previous cycle and the activity which flows into the ventricle from the atrium during diastole of the previous cycle. This relation can be written:

$$Q_{n+1} = Q_n(1 - EF) + Q_{An}$$

where

Q_n = total LV end-diastolic activity present at the beginning of cardiac cycle n .

Q_{n+1} = total LV end-diastolic activity present at the beginning of cardiac cycle $n + 1$.

Q_{An} = amount of new activity entering the LV from the atrium during diastole of cardiac cycle n .

EF = (total) LVEF.

The first term on the right side of the above equation is just the amount of activity present in the ventricle at end systole of cycle n , while the second term is the amount of new activity added to the ventricle due to inflow from the atrium during cycle n . We can now use this recursion relation to compute LV end-diastolic activity in every cardiac cycle if we begin with the first cycle.

In the first cycle, Q_1 is zero since no activity has yet entered the ventricle. The activity present in the ventricle at the beginning of cycle 2 is, by the recursion relation:

$$Q_2 = Q_{A1}$$

and at the beginning of cycle 3:

$$\begin{aligned} Q_3 &= Q_2(1-EF) + Q_{A2} \\ &= Q_{A1}(1-EF) + Q_{A2} \end{aligned}$$

and at the beginning of cycle 4:

$$Q_4 = Q_3(1-EF) + Q_{A3}$$

$$Q_4 = (Q_{A1}(1-EF) + Q_{A2})(1-EF) + Q_{A3}$$

and so on.

These expressions may be written in a more useful form:

$$\begin{aligned} Q_2 &= Q_{A1} \\ Q_3 &= Q_{A1}(1 - EF) + Q_{A2} \\ Q_4 &= Q_{A1}(1 - EF)^2 + Q_{A2}(1 - EF) + Q_{A3} \\ &\vdots \\ &\vdots \end{aligned}$$

Cumulative LV end-diastolic activity for the entire bolus transit is just the sum of these Q_s :

$$\begin{aligned} \text{Cumulative Activity} &= Q_2 + Q_3 + Q_4 + \dots = \\ &= Q_{A1}(1 + (1-EF) + (1-EF)^2 + (1-EF)^3 + \dots) \\ &+ Q_{A2}(1 + (1-EF) + (1-EF)^2 + \dots) \\ &+ Q_{A3}(1 + (1-EF) + \dots) \\ &+ Q_{A4}(1 + \dots) \\ &+ \dots \end{aligned}$$

It is clear that if no part of the bolus remains in the ventricle, i.e., if the transit is observed for an infinite period of time, then each of the terms Q_{An} will be multiplied by the same infinite series:

$$(1 + (1-EF) + (1-EF)^2 + (1-EF)^3 + (1-EF)^4 + \dots)$$

so that the cumulative activity becomes the product of two sums:

$$\text{Cumulative Activity} = (Q_{A1} + Q_{A2} + Q_{A3} + Q_{A4} + \dots)(1 + X + X^2 + X^3 + X^4 + \dots),$$

where $X = (1-EF)$.

The first term of this product is just the sum of all the activity that passes through the LV, namely the injected dose Q . The second term of the product is the infinite series representation of the fraction $1/(1-X)$. Making the appropriate substitutions gives:

$$\text{Cumulative Activity} = Q/EF.$$

Several aspects of this derivation deserve comment. First, note that this result is independent of where the bolus is introduced into the circulation or what the pattern of injection might be. We would obtain the same result for an intraventricular injection, intravenous injection or an injection made up of several discontinuous bursts of activity. The reason for this is that the Q_{An} appear as a sum which, in this instance, has a constant value equal to the injected dose. Since the sum is constant, the relative magnitudes of the Q_{An} , which determine the bolus "shape", can be adjusted at will, some Q_{An} even being zero, without changing the expression for cumulative activity. Note, however, that for this to be true each Q_{An} must be multiplied by the same infinite series. In order for this to happen, the entire bolus transit must be observed until negligible levels of activity remain in the LV. If, for example, recirculation requires data collection to be terminated before washout is complete, each Q_{An} will not be multiplied by the same infinite series. The cumulative activity will become instead a weighted average of the Q_{An} that will depend, to a greater or lesser degree, on both bolus shape and EF. In such cases, where data collection from the initial bolus transit must be

stopped before the transit is complete, the derived expression for cumulative activity will overestimate the actual first-pass cumulative activity and Equation 1 will overestimate the actual gated equilibrium equivalence time.

Finally, it can be shown that if valvular regurgitation is present, the same expression for cumulative activity holds if the total EF is everywhere replaced by the forward EF, EF_f , and the total cardiac output is replaced everywhere by the forward cardiac output. This substitution is valid only if it is assumed that a constant fraction of the activity ejected from the ventricle during systole fully returns to the ventricle during diastole of the same beat.

Definitions of Symbols Used in the Text

Q	= total injected tracer dose (mCi).
EF	= total EF (dimensionless).
t	= duration of end-diastolic image (sec).
e	= detection efficiency for LV activity (cps/mCi).
NF	= net LV end-diastolic counts for the first-pass method (dimensionless).
NE	= net LV end-diastolic counts for the gated equilibrium method (dimensionless).
V	= total blood volume (liters).
V_{ed}	= LV end-diastolic volume (liters).
M	= number of cardiac cycles recorded in gated equilibrium study (dimensionless).
HR	= heart rate (min ⁻¹).
T	= total gated equilibrium imaging time (min).
CO	= total cardiac output (liters/min).
MTT	= mean transit time (min).
S_F	= relative standard deviation of net LV end-diastolic counts for the first-pass method (dimensionless).
S_E	= relative standard deviation of net LV end-diastolic counts for the gated equilibrium method (dimensionless).
f	= ratio of relative standard deviation of net LV end-diastolic counts for the first-pass method to the relative standard deviation of net LV end-diastolic counts for the gated equilibrium method (dimensionless), i.e., S_F/S_E .

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SELF-STUDY TEST

True statements concerning bacterial overgrowth and the ¹⁴C-xylose breath test include which of the following?

14. Normal host tissue metabolism can result in high levels of CO₂ gas production.
15. Xylose is catabolized only by Gram-negative anaerobic bacteria.

16. Administration of 1 g of xylose normally increases endogenous CO₂ output by about 30% over basal levels.
17. Xylose is absorbed primarily in the small bowel.
18. A negative culture from a small bowel biopsy excludes bacterial overgrowth.

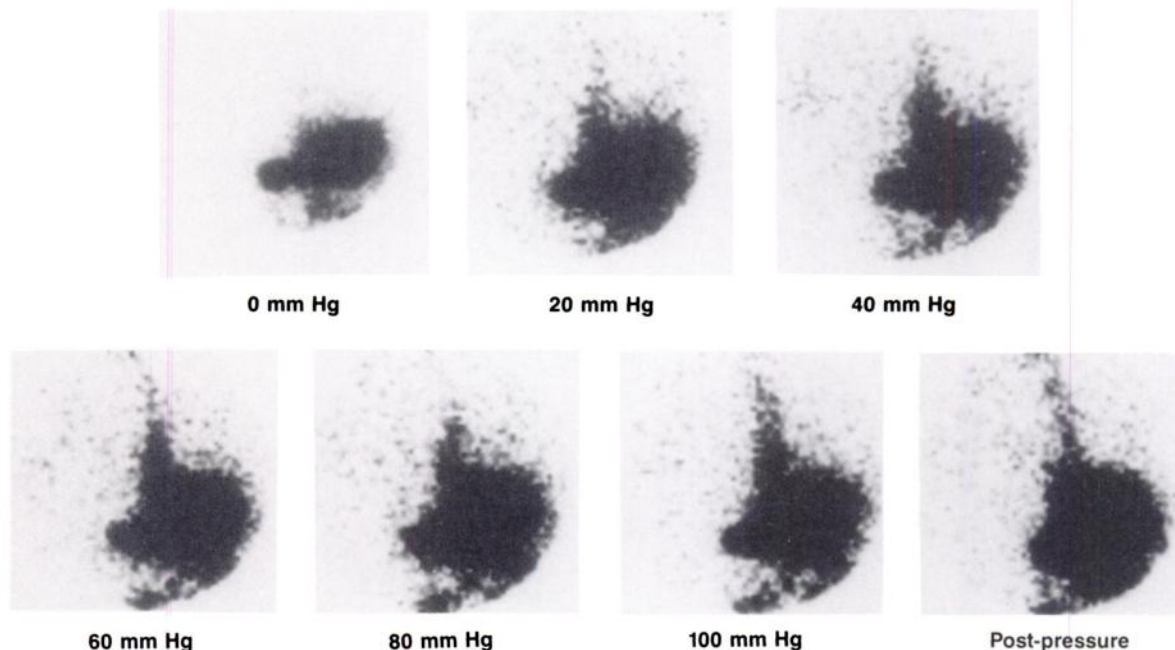


Figure 3

SELF-STUDY TEST

Gastrointestinal Nuclear Medicine

ANSWERS

ITEMS 1-4: Gastroesophageal Transit and Reflux

ANSWERS: 1, T; 2, F; 3, T; 4, F

Gastroesophageal reflux is clearly present (Fig. 3), and both the transit curves and "global" clearance curve indicate the presence of a mild motility disorder, which is characteristic of reflux esophagitis.

Although there is some apparent dilatation in the region of the distal esophagus, this can be explained by the anatomic finding of a hiatal hernia. A diagnosis of esophagitis with stricture should not be made on the basis of gastroesophageal scintigraphy and should be made either

from a barium swallow or at the time of endoscopy. The transit curves derived from the upper, middle, and lower thirds of the esophagus (Fig. 1) show retention in the distal esophagus. However, there is also a diffuse esophageal motility disorder, evidenced by the delay in transit of the bolus peak throughout the entire esophagus. Examination of the multiple-swallow emptying curve for the esophagus (Fig. 2) shows a pattern typical of esophagitis, with initially delayed clearance but eventual emptying of the esophagus after approximately 15 swallows. In a stric-

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