The inclusion of Compton scattering effects in the reconstruction matrix does present several practical problems. First, for a one-source-slice to one-projection-slice matrix such as that considered here, about 95% of the matrix elements are non-zero when scatter effects are considered, compared with only about 5% when scatter effects are ignored. If scatter effects are not included in the matrix, data compression techniques can be employed to reduce the memory requirements of the matrix, perhaps by a factor of ten, say from 30 megabytes to 3. Furthermore, as discussed in the Methods section, fully realistic treatments of scatter will require matrix elements specifying the probability of photons that were emitted from a given source slice (that is, a given y value) being detected in a different projection slice. Such a matrix would probably be impractical since it would be many times larger than the 30-megabyte matrix considered here. On the other hand, more compact representations of this scatter information may be possible. Scatter introduces a small but non-zero probability for a photon emitted from a given source location being accepted at almost any position on the detector. This probability varies slowly as a function of source location and detection location and therefore could perhaps be stored using coarse source and projection grids; alternatively, one might store the coefficients of low-order polynomial fits to these probabilities. Second, methods for obtaining, in an acceptable amount of computation time, the scattered-photon detection probabilities of a specific patient have not been worked out. Research on these problems is underway. This paper examines the degree of and the nature of the improvements in image quality that can be expected from these efforts.

## ACKNOWLEDGMENT

This work was supported by NIH grant CA46856.

## REFERENCES

1. Hamill JJ, Devito RP. Scatter reduction with energy-weighted acquisition. *IEEE Trans Nucl Sci* 1989;36:1334–1339.
2. Gagnon D, Todd-Pokropek A, Arsenault A, Dupras G. Introduction to holospectral imaging in nuclear medicine for scatter subtraction. *IEEE Trans Med Imag* 1980;8:245–250.
3. Gilardi MC, Bettinardi V, Todd-Pokropek A, Milanesi L, Fazio F. Assessment and comparison of three scatter correction techniques in single photon emission computed tomography. *J Nucl Med* 1988;29:1971–1979.
4. Msaki P, Axelsson B, Dahl CM, Larsson SA. Generalized scatter correction method in SPECT using point scatter distribution functions. *J Nucl Med* 1987;28:1861–1869.
5. Jaszczak RJ, Floyd CE, Coleman RE. Scatter compensation techniques for SPECT. *IEEE Trans Nucl Sci* 1985;32:786–793.
6. Floyd CE, Jaszczak RJ, Greer KL, Coleman RE. Deconvolution of Compton scatter in SPECT. *J Nucl Med* 1985;26:403–408.
7. Axelsson B, Msaki P, Israelsson A. Subtraction of Compton-scattered photons in single-photon emission computerized tomography. *J Nucl Med* 1984;25:490–494.
8. Jaszczak RJ, Greer KL, Floyd CE, Harris CC, Coleman RE. Improved SPECT quantification using compensation for scattered photons. *J Nucl Med* 1984;25:893–900.
9. Llacer J, Veklerov E. Feasible images and practical stopping rules for iterative algorithms in emission tomography. *IEEE Trans Med Imag* 1989;8:186–193.
10. Snyder DL, Miller MI, Thomas LJ Jr, Politte DG. Noise and edge artifacts in maximum-likelihood reconstructions for emission tomography. *IEEE Trans Med Imag* 1987;6:228–238
11. Lange K, Carson R. EM reconstruction algorithms for emission and transmission tomography. *J Comput Assist Tomogr* 1984;8:306–316.
12. Shepp LA, Vardi Y. Maximum likelihood reconstruction for emission tomography. *IEEE Trans Med Imag* 1982;1:113–122.
13. Dempster AP, Laird NM, Rubin DB. Maximum likelihood from incomplete data via the EM algorithm. *J R Stat Soc B* 1977;39:1–38.
14. Kemp M. Maximum entropy reconstructions in emission tomography. *Medical Radionuclide Imaging* 1980;1:313–323.
15. Liang Z, Jaszczak RJ, Greer KL. On Bayesian image reconstruction from projections: Uniform and nonuniform *a priori* source information. *IEEE Trans Med Imag* 1989;8:227–235.
16. Geman S, McClure DE. Statistical methods for tomographic image reconstruction. *Proc 46th ISI* 1987;52.
17. Levitan E, Herman GT. A maximum *a posteriori* probability expectation maximization algorithm for image reconstruction in emission tomography. *IEEE Trans Med Imag* 1987;6:185–192.
18. Floyd CE, Jaszczak RJ, Greer KL, Coleman RE. Inverse Monte Carlo as a unified reconstruction algorithm for ECT. *J Nucl Med* 1986;27:1577–1585.
19. Manglos SH, Floyd CE, Jaszczak RJ, Greer KL, Harris CC, Coleman RE. Experimentally measured scatter fractions and energy spectra as a test of Monte Carlo simulations. *Phys Med Biol* 1987;32:335–343.
20. Beck JW, Jaszczak RJ, Coleman RE, Starmer CF, Nolte LW. Analysis of SPECT including scatter and attenuation using sophisticated Monte Carlo modeling methods. *IEEE Trans Nucl Sci* 1982;29:506–511.
21. Wagner RF. Decision theory and the detail signal-to-noise ratio of Otto Schade. *Photo Sci Eng* 1978;22:41–46.
22. Rose A. *Vision: human and electronic.* New York: Plenum Press; 1973.

**EDITORIAL**

# Correction for Patient Compton Scattering—Current Status

Methods of correcting for the physical fact of Compton scattering of gamma rays within a patient with subsequent detection by an Anger camera are always extra work. The justifications for this extra work are (1) anticipated improvement of contrast in the image and (2) the potential for accurate quantification if the attenuation correction is also correct.

Techniques for Compton-scatter correction can be classified as pre-, during- or post-reconstruction. The extra work involved in the method is not directly related to the type. Pre-reconstruction methods include those of Gagnon et al. (*1*) and Koral et al. (*2*), which require acquisition of separate energy spectra for individual locations on the face of the Anger camera. The early one-dimensional projection convolution followed by subtraction of Axelsson et al. (*3*) and the later two-dimensional version by Msaki et al. (*4*) are also pre-reconstruction methods.

During-reconstruction methods are represented by (1) the true de-

Received Mar. 7, 1991; accepted Mar. 7, 1991.
For reprints contact: Kenneth F. Koral, Division of Nuclear Medicine, University of Michigan Medical School, Ann Arbor, MI 48109-0552.

convolution method of Floyd et al. (*5*) and (2) the expectation-maximization- (EM) with-scattering method of Bowsher and Floyd (*6*) reported on in this issue of the *Journal* and previously referred to as inverse Monte-Carlo reconstruction (*7*).

Post-reconstruction methods include the image-subtraction mode of the dual-energy-window method introduced to SPECT by Jaszczak et al. (*8*). In this mode, a separately-reconstructed image is subtracted from the normal tomographic slice to effect scatter correction. The projection-subtraction mode for this technique, on the other hand, falls under the pre-reconstruction heading (*9*).

As far as commercial clinical implementation, three techniques are, or perhaps soon, will be available. First of all, the possibility of a two-window tomographic acquisition has been provided for by at least one company (Siemens Medical Systems, Inc., Hoffman Estates, IL). Second, an on-line energy-dependent method known as WAM (Siemens) is available and has been investigated (*10*). In my opinion, the quantitative accuracy of this method did not motivate its original introduction and has not as yet been established. Lastly, another company (Elscint, Inc., Boston, MA) is looking into acquiring individual energy spectra as the basis for correction.

The new publication on the EM-with-scattering algorithm by Bowsher and Floyd emphasizes the maximum-likelihood, expectation maximization algorithm but Monte Carlo simulation is used, as before, to find the probabilities (also called weights) for each pixel in the image to have contributed to a particular projection element. It is the need for this latter calculation which is probably the weakest point of the method—more on this at the end of this editorial. It also clearly places the technique among those requiring considerably more work than in a normal SPECT reconstruction.

The algorithm itself is quite elegant in that all of the great variety of possibilities for transmission or scattering of gamma rays are taken into account. For instance, if there are 100 counts in a given projection element and that element has the relative probability of 0.001 of being reached from a laterally displaced pixel due to Compton scatters, then that pixel is likely to have a reconstruction strength of 0.1, according to this projection element. A non-displaced pixel facing little attenuation and having the larger relative probability of 0.1 is likely to have a strength of 10.0. All other pixels in the slice from which a gamma ray can originate and be received at the projection element within the energy window are also included with their own relative probability.

In their article, Bowsher and Floyd have done a very comprehensive job of investigating many of the important parameters of their subject in terms of several meaningful statistical measures. They include the effects of noise at three different count levels. They also present a comparison of accounting for scatter in the probability matrix of the reconstruction with not accounting for it.

On the other hand, the authors have chosen to present all their results in terms of lesion contrast, defined as the ratio of background minus signal over background. This choice has two consequences: (1) the quoted results are dependent on the method for background calculation and (2) absolute quantification of the lesion activity with the EM algorithm including scatter is not directly investigated. The method for calculating background using "the average pixel value in a ring surrounding, and extending a few cm beyond, the lesion" is, however, quite reasonable.

Also, the authors have simulated phantoms in which the activity does not vary with distance along the axis of rotation. Given that they wish to carry out 500 iterations and do an ensemble of 20 noisy cases for each data point, their restriction is understandable.

By comparing reconstructions from an ensemble of noisy data to that from noise-free data, Bowsher and Floyd conclude that noise affects the average bias in contrast only weakly or not at all. They also state that, generally, the bias averaged over the ensemble of 20 cases decreases with the number of iterations up to 500 (as we would expect) but that the standard deviation of the contrast, $\Delta C$, increases.

I would have preferred that the authors plot root-mean-square (RMS) error in the contrast against iteration number in Figure 4 rather than the average contract, $\bar{C}$, and $\Delta C$. However, it is true that (1) the bias in contrast can be simply calculated from $\bar{C}$ and the true contrast, $C_T$, by the equation bias = $C_T$-$\bar{C}$ and (2) the RMS error can then be obtained from $\sqrt{\Delta C^2 + \text{bias}^2}$. The reason for my preference is that the RMS statistic is a combination of the bias and standard deviation and, thus, (1) summarizes both to some extent and (2) gives a single figure of merit in the noisy situation. One can then obtain an optimum number of iterations, at least as judged by this figure of merit.

For the $C_T = 0.5$ (cold) lesion, off the cylinder center by 8 cm, with 200,000 count data, Bowsher and Floyd do point out that the RMS error in contrast (which combines bias and fluctuation as discussed above) does have an optimum at iteration 38. The existence of this optimum iteration number means that, even in this method where scatter is being taken into account, life isn't simple since more than 38 iterations are not better than exactly 38. Thus, in using the method, one cannot plan to iterate as long as is practicable and get the best answer given the time available. Rather, for any particular geometry, there appears to be a number of iterations one should approach but not exceed. This problem can, of course, occur with other iterative algorithms as well.

Perhaps the major question about the future of this algorithm concerns how the Monte Carlo calculation to obtain the probabilities is to be carried out in the case of patients. Here there are two problems: (1) the object does vary in the dimension along the axis of rotation and (2) a model for the patient is not easily available.

The authors discuss the matter of the memory storage requirement for their technique when the object varies along the axis of rotation. This is presumably solvable with advances in the field.

The second problem of a patient model "has not been worked out" according to Bowsher and Floyd. I have had experience with superimposing individual slices of a patient CT scan on those of a SPECT reconstruction (*11*). An algorithm to obtain attenuation-coefficient maps from such CT slices is under investigation in my research work. An extension to obtaining Compton-scatter cross sections might be practicable. Future work on implementing the EM algorithm with scatter included might proceed in this direction. Probably that is the next step in the reasonable development of this inherently elegant but practically difficult approach. Success or failure in this area will likely determine the possible entrance of this technique into the competition of commercial, clinically implemented techniques.

**Kenneth F. Koral**
*University of Michigan*
*Medical School*
*Ann Arbor, Michigan*

## REFERENCES

1. Gagnon D, Todd-Pokropek A, Arsenault A. et al. Introduction to holospectral imaging in nuclear medicine for scatter subtraction. *IEEE Trans Med Imag* 1989;8:245–250.
2. Wang X, Koral KF, Clinthorne NH, Rogers WL, Floyd Jr CE, Jaszczak RJ. Effect of noise, order and range in fitting the photopeak region of local, Anger-camera energy spectra. *Nucl Instr Methods in Phys Res* 1990;A299:548–553.
3. Axelsson B, Msaki P, Israelsson A. Subtraction of Compton-scattered photons in single-photon emission computed tomography. *J Nucl Med* 1984;25:490–494.
4. Msaki P, Axelsson B, Dahl CM, Larsson SA. Generalized scatter correction in SPECT using point scatter distribution functions. *J Nucl Med* 1987;28:1861–1869.
5. Floyd CE, Jaszczak RJ, Greer KL, Coleman RE. Deconvolution of Compton-scatter in SPECT. *J Nucl Med* 1986;26:403–408.
6. Bowsher JE, Floyd Jr CE. Treatment of Compton scattering in maximum-likelihood, expectation-maximization reconstruction of SPECT images. *J Nucl Med* 1991;32:1285–1291.
7. Floyd CE, Jaszczak RJ, Greer KL, Coleman RE. Inverse Monte Carlo as a unified reconstruction algorithm for ECT. *J Nucl Med* 1986;27:1577–1585.
8. Jaszczak RJ, Greer KL, Floyd Jr CE, Harris CC, Coleman RE. Improved SPECT quantification using compensation for scattered photons. *J Nucl Med* 1984;25:893–900.
9. Koral KF, Swailem FM, Buchbinder S, Clinthorne NH, Rogers WL, Tsui BMW. SPECT dual-energy-window Compton correction: scatter multiplier required for quantification. *J Nucl Med* 1990;31:90–98.
10. DeVito RP, Hamill JJ. Determination of weighting functions for energy-weighted acquisition. *J Nucl Med* 1991;32:343–349.
11. Koral KF, Ten Haken RK, McShan DL, et al. Superimposition of SPECT and CT images and transfer of RoI for quantification [Abstract]. *J Nucl Med* 1990;31:872.