Since a unique sphere is delineated by the ratio of its volume, V, and diameter, D, the authors use their Equation 4 (as corrected), $D = (6M^2R/\pi)^{1/2}$, to calculate D, the diameter (and hence the volume) of a presumed sphere, where R is obtained from the total counts of the ellipsoid as viewed along the x-axis (LAO projection) and the maximum pixel counts from a reference volume along the long-axis corrected by M^2, the calibration factor for pixel area. An exact equivalent expression necessitating neither total counts nor maximum pixel counts and using the same assumption of a spherical LV may be obtained geometrically as follows:

Let P = total number of pixels of the circle in the plane of the short-axis, S. Its area K_c, as seen from the LAO projection, is given by:

$$K_c = PM^2 = \pi \left(\frac{S}{2}\right)^2 \qquad \text{Eq. 1}$$

so that

$$\frac{S}{2} = M \left(\frac{P}{\pi}\right)^{1/2}. \qquad \text{Eq. 2}$$

Since a sphere is assumed, S is the diameter, and

$$V_s = \frac{4}{3} \pi \left(\frac{S}{2}\right)^3 \qquad \text{Eq. 3}$$

$$V_s = 0.752\ M^3\ P^{3/2} \qquad \text{Eq. 4}$$

As an example, let L = 8, S = 6, and M = 0.1 cm. Then, the correct volume of the prolate ellipsoid is

$$V_e = \frac{\pi}{6} LS^2 = 151\ \text{cm}^3.$$

By the authors' method, if each voxel of volume M^3 contains one count, then

$$C_t = 151{,}000$$

and

$$C_r = 80$$

$$R = \frac{151{,}000}{80} = 1887$$

and by the authors' Equation 5

$$V_t = 1.382(0.1)^3(1887)^{3/2} = 113\ \text{cm}^3.$$

By my method, the area $K_c = \pi \left(\frac{S}{2}\right)^2 = 28.27\ \text{cm}^2$ and from Equation 1

$$P = \frac{28.27}{0.01} = 2827.$$

Equation 4 yields

$$V_s = 0.752(0.1)^3(2827)^{3/2} = 113\ \text{cm}^3$$

identical to the value obtained by the authors' method.

Parenthetically, the resultant error in this example as compared to the volume of the prolate ellipsoid is (151 − 113)/(151) = 25%.

Thus, by knowing the number of pixels, P, and the size of the pixel, M, one may calculate the volume of the sphere without even determining the counts.

Since it can be shown that $R = \frac{4}{6}P$, the authors' Equation 5 and my Equation 4 are exactly equivalent, thus showing that the authors' method is a geometric one. In the method I employ, the area of the LV in the LAO projection is used to find the volume rather than a direct measurement of S, since in practice the projection of the LV is rarely circular. This direct geometric method has the advantage that no assumptions are made regarding either attenuation or the equivalency of the factor K in the authors' Equations 1 and 2.

It is not surprising that the authors obtained good correlation between their method, a *geometric* model which considers the LV a sphere (i.e., an ellipsoid of 0 eccentricity) with the contrast angiocardiographic method which geometrically models the LV as a prolate ellipsoid. As the eccentricity of the ellipsoid approaches zero, the authors' assumptions improve. If a geometric method is to be used at all, the standard prolate ellipsoid model should be employed.

The count-based nongeometric methods avoid *all* assumptions as to the shape of the LV and also give high correlation coefficients with contrast angiocardiographic methods (*4*); the values obtained are probably more realistic.

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REPLY: Dr. Nusynowitz is concerned that we have misrepresented our approach to a count-based chamber volume determination as non geometric. He correctly points out the fact that we assume a spherical model of the left ventricle (LV) in the LAO view in order to establish the relationship between counts (N_{max}) and a reference volume (M^2d). Given that relationship, we can then calculate the volume of any chamber, regardless of shape and, in fact, the LV volume measurement comes from the counts in the LV at end-diastole and the relationship described above. The reference volume approach requires some calculable volume within the image data to define the counts-to-volume relationship. However, no specific assumptions about the geometry of the chamber to be measured are necessary.

TABLE 1
Values for Eccentricities in Equation 1

E	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.84	0.9	0.99
θ deg	54.7	54.7	54.5	54.1	53.5	52.8	51.7	50.2	47.8	46.5	43.6	30.4

As it turns out, we use a spherical model of the LV to calculate our reference volume, the justification of which is as follows.

It can be shown that any ellipsoid of revolution can be viewed as a perfect sphere provided that the angle of view, θ, defined as the angle between a line perpendicular to the plane of the collimator and the long-axis of the ellipsoid has the proper value. The value of θ, which guarantees that any ellipsoid of eccentricity, E, will be sampled as a perfect sphere whose volume is exactly equal to the volume of the ellipsoid, can be shown to be given by the following equation:

$$\theta = \tan^{-1} \sqrt{[1 + (1 - E^2)^{1/3}](1 - E^2)^{1/3}}. \qquad \text{Eq. 1}$$

When viewed from the angle, θ, an ellipsoid of eccentricity, E, is observed with count profiles that are indistinguishable from a sphere with the same volume as the ellipsoid. Table 1 lists the values of θ from Equation 1 for eccentricities, E, from 0.01 to 0.99.

The LAO view preferred for gated blood-pool studies is obtained by optimizing the anatomic separation between the RV and LV. This requirement generally results in a value of θ between 40 to 50 degrees. The eccentricity of the LV can be shown from biplane contrast ventriculography studies to be 0.77 ± 0.07 at end-diastole. The data in Table 1 show that with this range of eccentricities, the LV behaves very nearly like a sphere for values of θ from 50.2° to 46.5°. This is a sufficiently close match to the actual sampling angle in the LAO view used in gated studies to treat the count profiles as if they originate from a spherical distribution. The example given by Nusynowitz can serve to illustrate the sphericalization of an ellipsoid by proper viewing angle. The ellipsoid in the Nusynowitz letter is defined with a major axis, L, of 8 cm and a minor axis, S, of 6 cm, the volume is 151 cm^3, M = 0.1 cm, and the eccentricity is E = 0.661. Equation 1 allows us to calculate the angle $\theta = 50.8°$, which will sample this ellipsoid as if it is a sphere of diameter, $d = (LS^2)^{1/3} = 6.6$ cm. The constant of proportionality, K, between counts and volume is assumed as 1000 cts/cm^3. The sampled volume of the reference volume is $V_{ref} = dM^2 = 6.6 \times 0.01 = 0.066$ cm^3, corresponding to the region centered on effective diameter, d. The counts from

$$V_{ref} = V_{ref}K = 0.066 \times 1000 = 66,$$

$$C_t = 151{,}000 \therefore R = \frac{C_t}{C_r} = \frac{151{,}000}{66} = 2288,$$

$$\text{and } V_t = 1.382(0.1)^3(2288)^{3/2} = 151 \text{ cm}^3,$$

the exact volume of the ellipsoid. Nusynowitz would be numerically correct if sampling occurred at $\theta = 0$ degrees, but in the typical LAO view, this is not the case and θ is more nearly equal to the sphericalization angle.

The eccentricity of the LV quite fortuitously provides a nearly perfect match between the clinically favored LAO viewing angle and the sphericalization angle, θ. This happy circumstance, however, should not obscure the fact that the "count-proportional reference volume" method is indeed a geometry-independent method. The strategy that should be used with the count method is to find a reasonable reference volume within the observed data. Frequently, the reference volume is found in any region which has a circular cross-section. The aorta and LV provide numerous opportunities in many views to select an adequate reference volume. For example, the LV in the anterior view presents a number of cross-sections along the short-axis from apex to base which can reasonably be assumed to be nearly circular. In this case, a single row of pixels is flagged along the short-axis. The volume of the cross-section is $V_{row} = \frac{\pi}{4} S^2M$ and the count from this volume is $C_{row} = V_{row}K = \frac{\pi}{4} S^2MK$, where S is the diameter of the ventricle in that cross-section, M is the length of a side of a pixel, and K is ct/cm^3 for the tracer concentration in the ventricle. The maximum count in this row of pixels, C_{max}, will originate from the volume region, SM^2, centered on the diameter, S, and $C_{max} = SM^2K$. Combining these two measurements and solving for K, we obtain:

$$K = \frac{\pi}{4} \frac{C_{max}^2}{M^3 C_{row}}. \qquad \text{Eq. 2}$$

The total counts, C_t, from the entire ventricle are $C_t = V_tK$. In the previous example presented by Nusynowitz, sampling along the circular cross-section, S, with M = 0.1 cm, would yield:

$$V_{row} = \frac{\pi}{4} S^2M = 2.827 \text{ cm}^3$$

$$V_{max} = SM^2 = 0.06 \text{ cm}^3 \therefore C_{row} = 2827 \text{ cts}$$

$$C_{max} = 60 \text{ cts}$$

$$K = \frac{\pi}{4} \frac{60 \times 60}{(0.1)^3 2827} = 1000 \text{ cts/ml}$$

$$V_{tot} = 151 \text{ cm}^3 \therefore C_t = 151{,}000$$

$$V = \frac{C_t}{K} = \frac{151{,}000}{1{,}000} = 151 \text{ cm}^3$$

Hence, $V = \frac{C_t}{K}$, where K is obtained from Equation 2. The value of K can be obtained from more than one cross-section and averaged. This powerful technique was not used in our paper "Left Ventricular Volume Calculation Using a Count-Based Ratio Method Applied to Multigated Radionuclide Angiography" because it requires rotation of the data to guarantee that reference cross-sections lie along rows or columns of the pixel matrix. This requirement is necessitated because the quantity, M, must correspond to the length of the side of a pixel. The computer capability available today should allow proper data rotation to permit the use of circular cross-sections as reference volumes available in many radionuclide measurements. The determination of K in cts/cm^3 is the important result. The "count-proportional reference volume method" uses a reference volume imbedded in the data as opposed to the approach proposed by Nusynowitz in his letter and in Reference 1, which requires blood sampling to provide a reference volume.

REFERENCE

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