Our article was not intended to be a blow-by-blow account of the resolution of the problem. Our goal was to alert colleagues to a situation that could lead to inappropriate patient care and to offer our suggestion for detecting such a problem and preventing image misinterpretation. Although, we may have been the one institution to report this problem to the manufacturer, personal communications from other institutions have assured us that we were not the only institution to note the malfunction.

We thank Dr. Bernstein for his interest in our publication and for clarifying the cause of the image switching malfunction noted.

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Statistical Artifact in DPA Measurements at Low Count Rates

TO THE EDITOR: I would like to offer a possible explanation for the apparent increase in bone mineral content (BMC) observed by DaCosta et al. (*1*) at low counting rates. The increase amounted to ~0.04 g/cm^2 for an aluminum "bone phantom" scanned in 24.5 cm of water, with a 0.3-Ci source and "narrow" (8-mm) detector collimation on a Lunar DP3 scanner. Although counts per pixel were not specified, the conditions described suggest that they were "small." A statistical artifact that occurs in low-count data may explain the effect.

The equation for computing BMC, BMD (g/cm^2), is given by (*2*):

$$BMD = CF\,(A_b - A_s), \quad (1)$$

where

$$A_b = R\,\ln(N_{hb}) - \ln(N_{lb}) \quad (2)$$

$$A_s = R\,\ln(N_{hs}) - \ln(N_{ls}) \quad (3)$$

$$R = u_{sl}/u_{sh} \quad (4)$$

$$CF = 1/(u_{bl} - R\,u_{bh}) \quad (5)$$

In the above equations, N_{hb} and N_{lb} are the high- and low-energy photon counts measured through bone, N_{hs} and N_{ls} are the counts measured in the soft-tissue baseline, u_{bh} and u_{bl} are the mass attenuation coefficients of bone mineral at the high- and low-photon energies, and u_{sh} and u_{sl} are the corresponding quantities for soft tissue.

In practice, A_b and A_s are measured at many points and averages are taken to compute the patient's average BMD. Random variations in count rate occur from point-to-point due to statistical fluctuations in source decay. Implicit in the averaging procedure is the assumption that the mean of the observed counts equals the "true counts" and that mean of the logarithm of the observed counts, N, is equal to the logarithm of the mean, m:

$$\langle \ln(N) \rangle = \ln(m). \quad (6)$$

This assumption generally is not valid, because a statistical fluctuation of one count below the mean causes a greater discrepancy in the logarithm than a fluctuation of one count above the mean. For example, ln(10) = 2.303, whereas the mean of ln(9), ln(10), and ln(11) is 2.299.

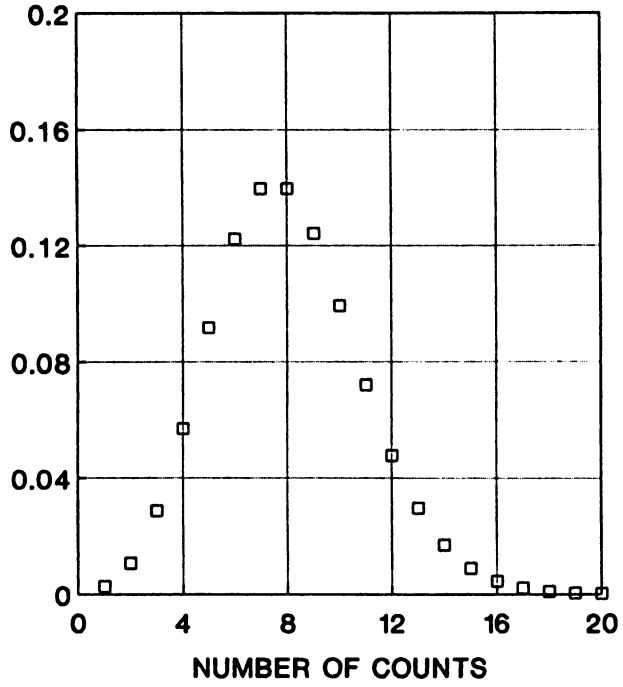

FIGURE 1
Poisson probability distribution for mean number of counts = 8.

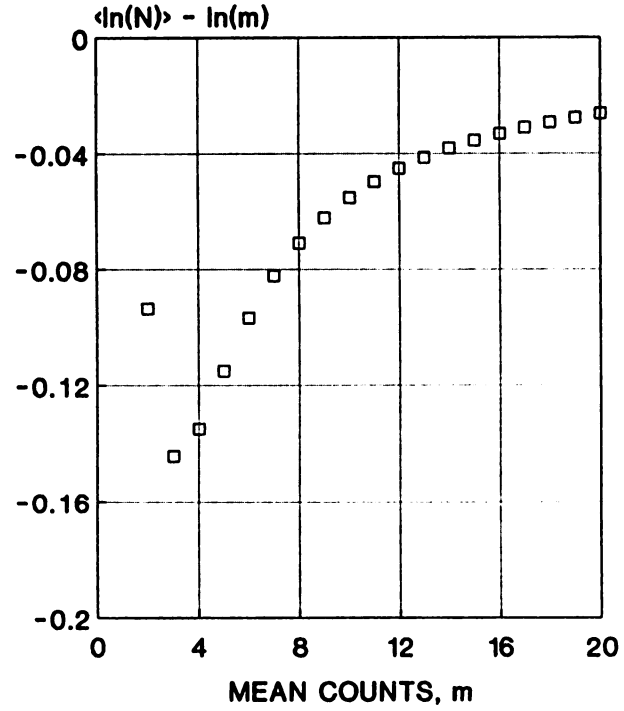

FIGURE 2
Difference between mean of logarithms vs. logarithm of mean for Poisson count distributions with different mean values, m.

A few additional exercises with a pocket calculator will demonstrate that the discrepancy for symmetric count distributions is not large except at relatively low count values. However, the situation becomes even worse at low count values, because the Poisson distribution is not symmetric. Specifically, the probability of obtaining a result smaller than the mean is greater than the probability of obtaining a result larger than the mean. This further skews the mean of the logarithm of observed counts, <ln(N)>, to smaller values. Figure 1 demonstrates this for a Poisson distribution having a mean of 8.

Figure 2 shows the discrepancy between the mean of the logarithm of observed counts versus the logarithm of the mean for Poisson distributions having different mean values, m. To compute the values for Figure 2, I assumed that observations with N = 0 would be set to N = 1, since ln(0) is undefined. Errors are obtained even for m = 100, but the discrepancy becomes large for m < 20.

Note also that all discrepancies, except for m = 1 (not shown), are negative. In Equations 1–3, the smallest count values generally are those for the low-energy bone counts, N_{lb}. Although all of the count values are affected by this statistical artifact, and to some extent cause offsetting errors in Equations 1–3, the largest effect occurs for low-channel bone counts. This would cause an *increase* in calculated BMD.

Figure 3 shows the effect of this statistical artifact versus low-channel baseline counts per pixel for BMD = 1 g/cm^2 in 25-cm muscle, including the effect of the statistical artifact on all of the count measurements in Equations 1–3. The computations for this figure used the following input data derived from the simulation model described in Ref. *2*: $N_{hs}/N_{ls} = 6$, $u_{lb} = 0.858$ cm^2/g, $u_{hb} = 0.201$ cm^2/g, R = 1.56, CF = 1.84.

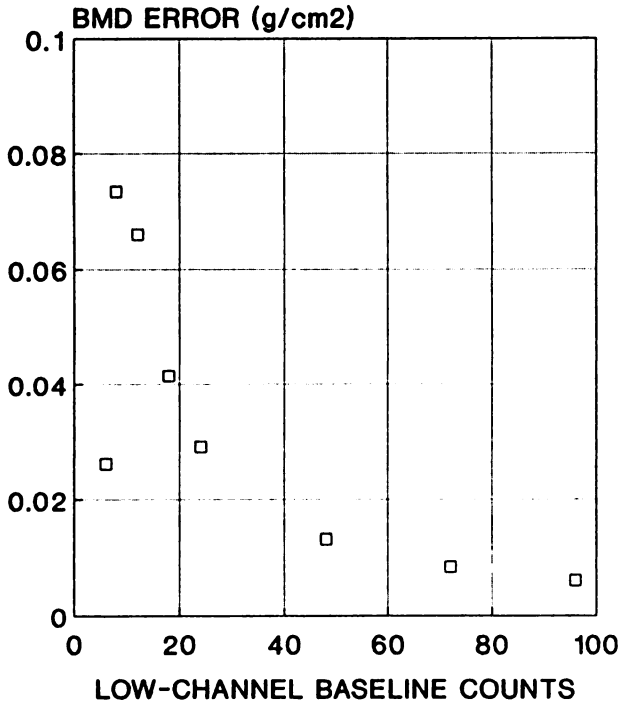

FIGURE 3
Error in computed BMD vs. low-channel soft-tissue baseline counts, calculated from simulation data (see text).

Figure 3 suggests that a marked increase in calculated BMD should be observed when low-channel baseline counts become smaller than ~50 counts per pixel. The effect actually observed may vary on different systems, depending on corrections that may be employed at low count rates. It also will vary with patient thickness and BMD, which affect the actual count values in Equations 1–3. An "exact" correction for the statistical artifact could be made using the data in Figure 2 to correct individual count data. Without such corrections, it would be advisable to avoid DPA measurements with counts per pixel less than ~50.

Note that the effects described here depend on counts per pixel, not source activity or attenuation. Furthermore, they occur in any measurement that includes logarithmic averaging of low-count data.

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Dual-Photon Absorptiometry: Depleted Sources Inappropriate in Obese Patients with Narrow Collimation

TO THE EDITOR: DaCosta et al. (*1*) recently demonstrated differences of bone mineral density (BMD) results associated with marked differences of source strength and attenuation thickness using gadolinium-153 (^{153}Gd) dual-photon absorptiometry. A small increase of BMD at low source activities (<0.3 Ci) does occur in thick patients (>20 cm) using older spine software on the Lunar DP3 scanner. The well-documented increase of ~0.02 to 0.04 g/cm^2 amounts to a 2%–4% increase at the typical BMD for elderly women of 1.0 g/cm^2 (*2–6*). This shift of spine results, due to a software bug, was corrected in later software versions (*4, 5*); it is not evident in femur scans. The unusual findings of DaCosta et al. using recent software may be due to their use of (a) 8-mm detector collimation, (b) a very depleted source, (c) two different sources for high- and low-activity determinations, and (d) a large thickness of a nonphysiologic attenuator.

Using 8-mm collimation rather than the standard 13-mm collimation reduces the count flux by over twofold. When using 8-mm collimation, sources can be used only to ~0.5 Ci, so the 0.3 Ci source used by DaCosta et al. was 6 mo beyond its useful life. Moreover, two different sources were used in testing. Nilas et al. (*6*) have shown that shifts of several percent can accompany use of different depleted ^{153}Gd sources, possibly as a consequence of contamination.

DaCosta et al. tested at a water thickness of 24.5 cm, which is equivalent to the attenuation seen in a patient 26-cm thick (15 cm of lean tissue and 11 cm of fat, which has 20% less linear attenuation than water). In our examination of several