Med Biol 1988; 33:969–973.
29. Shepp LA, Logan BF. The Fourier reconstruction of a head section. *IEEE Trans Nucl Sci* 1974; 21:21–43.
30. Chesler DA, Riederer SJ. Ripple suppression during reconstruction in transverse tomography. *Phys Med Biol* 1975; 20:632–636.
31. Grant DG. Tomosynthesis: a three-dimensional radiographic imaging technique. *IEEE Trans Biomed Eng* 1972; 19:20–28.
32. Rowland SW. Computer implementation of image reconstruction formulas. In: Herman GT, ed. *Image reconstruction from projections. Implementation and applications.* Heidelberg: Springer Verlag; 1979:9–79.
33. Mueller TM, Marcus ML, Ehrhardt JC, Chaudhuri T, Abboud FM. Limitations of thallium-201 myocardial perfusion scintigrams. *Circulation* 1976; 54:640–646.
34. Burow RD, Pond M, Schafer AW, Becker L. "Circumferential profiles: " a new method for computer analysis of thallium-201 myocardial perfusion images. *J Nucl Med* 1979; 20:771–777.
35. Bone D, Holmgren A, Svane B. Thallium tomography in the detection of myocardial perfusion defects. *Acta Med Scand Suppl* 1984; 694:109–119.
36. Dale S, Edholm P. Inherent limitations in ectomography. *IEEE Trans Med Imag* 1988; 7:165–172.
37. Schwartz JS, Ponto R, Carlyle P, Forstrom L, Cohn JN. Early redistribution of thallium-201 after temporary ischemia. *Circulation* 1978; 57:332–335.

EDITORIAL: Limited-Angle Tomography for the Nineties

This issue of *The Journal of Nuclear Medicine* includes two articles (*1,2*) on what many would regard as an outmoded technique, limited-angle tomography with a rotating slant-hole collimator (RSH). Is there a role for such methods in the age of sophisticated tomographic techniques such as SPECT and PET? Before attempting to answer that question, let us take a brief look at the historical and mathematical background.

HISTORICAL BACKGROUND

Tomography is one of the oldest ideas in radiology. The basic concept of selecting a plane of interest by relative motion of source and detector is apparently due to Bocage in 1921 (*3*), but the first practical demonstration was made by Vallebona in Italy in 1930 (*4*). Many ingenious variations on this theme were developed over the next half century, and many different names, such as planigraphy, stratigraphy and laminography, were applied. We shall refer to all of these methods collectively as *classical tomography.* The common ingredient in these methods is that they obtain depth discrimination by parallax.

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The advent of computed tomography in the seventies essentially led to the demise of classical tomography in diagnostic radiology (though a few clinical applications remain). Computed tomography (CT) is superior because it completely eliminates image information from planes other than the desired one (for the simple reason that the radiation is confined to that plane). By contrast, the classical methods merely blur the undesired planes, as in a microscope, but do not eliminate them. The out-of-focus structures reduce the contrast in the plane of interest and interfere with diagnosis. In this discussion, we shall use the terms computed tomography and classical tomography in a broad sense: classical tomography is any method that blurs undesired planes, while computed tomography eliminates them. It is clear that CT is preferred over classical methods in diagnostic radiology today.

In nuclear medicine, the evolution of tomography took a rather different course. The earliest tomographic system for nuclear medicine, introduced by Kuhl and Edwards (*5*) in 1963, was in fact a CT system (SPECT, in particular). Classical motion tomography was introduced to the field a little later, with the advent of the Anger tomoscanner (*6*) in 1966 and the Vanderbilt tomoscanner (*7*) in 1969. Many other methods soon followed, among them the RSH collimator, the seven-pinhole aperture, and a wide variety of coded apertures. For a good review of this field, see Koral (*8*).

MATHEMATICAL BACKGROUND

To this point we have not introduced the term *limited-angle tomography.* One definition of this term is by exclusion: a limited-angle tomographic system is one in which the data collection does not span the full range of projection angles needed for accurate image reconstruction. To apply this definition, we must of course specify the required full angular range. For parallel-beam projections confined to a plane, as in the first-generation CT scanners, the full angular range in projection angle is 180°. The easiest way to see this is to appeal to the projection-slice theorem, which states that each projection gives information about the Fourier transform of the object along one line through the center of the two-dimensional (2-D) Fourier plane. If projections are collected for all angles over a range of 180°, these lines sweep over the entire 2-D Fourier plane. Since an object is uniquely specified by its Fourier transform, it can thus be reconstructed unambiguously from projections over a continuous set of projections spanning 180°. Of course, errors are introduced by finite angular steps, noise in the data and detector limitations, but these effects are not our concern here.

Having defined the full angular range, we can now see the effect of a limited range. If the projection angles cover a range less than 180°, then there are two vee-shaped regions in the 2-D Fourier plane (see Fig. 1) that are never measured. Object Fourier components in these regions are lost, and artifacts appear in the reconstructions.

The problem gets more interesting in classical tomography or other methods where the radiation is not confined to a plane. Then the object is specified by its three-dimensional (3-D) Fourier transform, and the key question is what portion of the 3-D Fourier domain is sampled.

Consider a scintillation camera with a parallel-hole collimator viewing a 3-D activity distribution. The 3-D counterpart of the projection-slice theorem says that the 2-D Fourier transform of each projection is one plane through the 3-D Fourier transform of the object, with the normal to the plane parallel to the projection direction as defined by the collimator bores. If the camera and collimator rotate around the object as in ordinary SPECT, with the axis of rotation perpendicular to the projection direction, then the plane in Fourier space rotates around a line contained in the plane, and the entire 3-D Fourier space is swept out.

Another way to think of this case is to consider one stripe on the camera face. As the camera rotates around the patient, this stripe receives radiation from only one slice in the body, so the problem is mathematically equivalent to a parallel-beam CT scanner. Activity in that slice can be reconstructed from projections over 180°, and the full 3-D object can be represented as a set of parallel slices. With either viewpoint, it can be seen that a 180° rotation yields a complete data set for object reconstruction.

The situation is very different if the axis of rotation is not perpendicular to the projection direction. Rotating slant-hole collimators, the subject of the articles in this issue, provide an excellent example of this situation. Again, at each position of the collimator, a 2-D parallel projection of the 3-D object is measured, giving information about the object Fourier transform on a plane in 3-D Fourier space. In this case, however, the plane in Fourier space rotates about a line not contained in the plane and not all of the Fourier space is sampled.

The reader can easily demonstrate the effect by holding a pencil vertical with its point in contact with a sheet of cardboard; the cardboard is the plane in Fourier space, while the pencil is the axis of rotation. By the 3-D projection-slice theorem, the projection direction is always perpendicular to the cardboard. For the RSH collimator, the cardboard makes some fixed angle with the pencil and rotates around it. Consider a point in space near the horizontal plane through the pencil point. This point may not lie on the cardboard at first, but as the cardboard rotates, it will eventually intersect the point in question. On the other hand, consider a second point in space above the cardboard and near the pencil. This second point will never be intersected by the rotating cardboard, so no information about that point in the object Fourier transform will ever be recorded. A few moments of playing with this model should convince the reader that there is an entire double cone of points that will never be intersected by the cardboard. These are the famous *missing cones* in Fourier space (see Fig. 2). They are the 3-D analog of the vee-shaped regions described above.

The case of conventional SPECT is represented with the pencil model by letting the cardboard be initially parallel to the pencil and in contact with the shaft. (It helps to imagine the pencil as having an infinitesimal thickness.) Then, as the cardboard rotates around the pencil, all points in the 3-D space are eventually intersected. The cone angle of the missing cones shrinks to zero in this case, so all of Fourier space is sampled and a good reconstruction can be obtained.

RECONSTRUCTION ALGORITHMS

There is a huge literature on the limited-angle problem, both in 2-D and in 3-D. For a recent bibliography, see Rangayyan et al. (*9*). So much powerful mathematics has been brought to bear on this problem that the unwary reader of this literature might get the idea that it has been solved in some sense. There are, fortunately, a few general principles that we can fall back on in attempting to wade through the literature.

The first principle is that there is no magic filter. No amount of linear filtering can ever recover Fourier components that are lost in the data-collection process. A linear filter simply multiplies each Fourier component by some factor, but zero remains zero.

A corollary principle is that prior information about the object offers the only hope of ever filling in the missing cones. Prior information is what we know about the object before making the measurements. In medical imaging, we certainly know a priori that the object has a finite spatial extent; after all, the patient fits into our imaging system. This knowledge is referred to as a *support constraint*, the support being the spatial region into which the object is known to fit. We also know that an activity is inherently a non-negative number, and this knowledge is called (somewhat loosely) a positivity constraint. Other prior knowledge might include some statistical description of the spatial distribution or knowledge of the prior probability of disease.

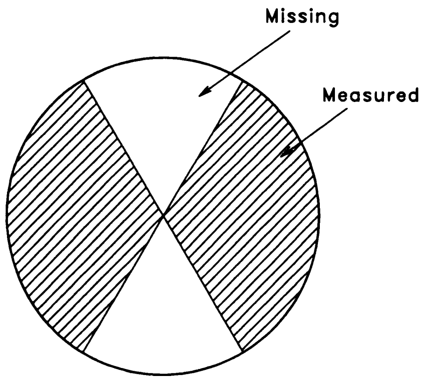

FIGURE 1
Illustration of the region in a 2-D Fourier plane for which measurements of the object Fourier transform are obtained with 2-D limited-angle tomography.

The support constraint is a very interesting one to mathematicians. It implies that the object Fourier transform is an *analytic function*, which for our purposes means that the complete function can be extrapolated from knowledge of it over a finite domain. In particular, if we know the Fourier transform over the measurement region in Figure 1 or 2, we can extrapolate it into the missing regions. There are straightforward linear algorithms for this extrapolation, which would seem to belie the first principle enunciated above. On closer examination, however, the principle holds; the extrapolation methods turn out to be incredibly sensitive to noise and therefore virtually useless in practice. The most convincing demonstration of this point is found in a classic, though difficult, paper by Davison (*10*).

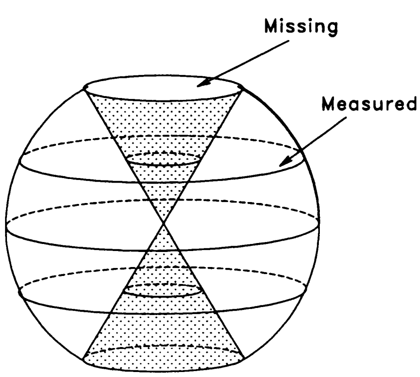

FIGURE 2
Illustration of the region in the 3-D Fourier space for which measurements of the object Fourier transform are obtained with classical 3-D limited-angle tomography.

The positivity constraint seems to be simultaneously more useful and more difficult to analyze than the support constraint. In some cases, positivity is capable of completely filling in the missing cones. As an extreme example, suppose the object consists of a single radioactive point. Attempting to reconstruct this object from limited-angle data with any linear filter or linear extrapolation method would inevitably introduce negative values, contrary to our prior knowledge. With nonlinear iterative algorithms that enforce agreement with the data and the prior knowledge, however, an excellent reconstruction will be obtained. There is, unfortunately, no mathematical theory that indicates just how far this approach can be pushed. We cannot yet quantify how effective a positivity constraint can be in compensating for missing data.

Another active area of research is the use of statistical prior knowledge in Bayesian reconstruction. There is anecdotal evidence that this method offers some advantages, but again little of a quantitative nature.

SOME CAVEATS

The reader familiar with the literature on limited-angle tomography will surely be able to cite references that seem to contradict the first principle above. Many linear algorithms have been published and shown to work splendidly on simulated data; somehow we never see the expected follow-up publications applying them to real clinical data.

There are several ways in which simulation can deviate markedly from reality. The most obvious is noise, which is inevitable with real data and often neglected in simulations. The linear extrapolation methods have already been mentioned as examples of algorithms that are extremely sensitive to noise. As another general principle, never trust a noise-free simulation.

Even if noise is properly included, however, the simulation may bear no resemblance to reality. The next point to suspect when attempting to evaluate a simulation study is the object model. A computer requires a discrete set of numbers, while real clinical objects are continuous objects in three dimensions. A common way of "discretizing" an object for simulations is to consider it to be made up of points or cubes. If a good reconstruction then results, it *might* mean only that the system is capable of imaging objects made up of points or cubes, while it would fail completely with real, continuous objects. Limited-angle tomography is especially susceptible to this pitfall, but the problem can be avoided by using very fine sampling or continuous mathematics in simulating the data. The principle: never trust any simulation that uses the same object model for producing the data as for the reconstruction.

Another potential pitfall is that simulation studies may build in prior knowledge or assumptions that are not valid with real objects. Examples include algorithms that require geometric transformations of the object or that restrict the object to a narrow range of gray levels. The algorithm may work if the postulated conditions are satisfied, but that is small consolation if they are impossible to satisfy in practice.

ECTOMOGRAPHY

In light of the principles enunciated above, how are we to evaluate the method of ectomography (*11*, *12*) favored by Dale and Bone in the work reported in this issue? Clearly, this algorithm works with real data, so it is immune from the criticism of simulations. It is, however, a linear filtering method, so it does not fill in the missing cones. Dale and Bone are well aware of this point and do not present ectomography as a general-purpose algorithm or a

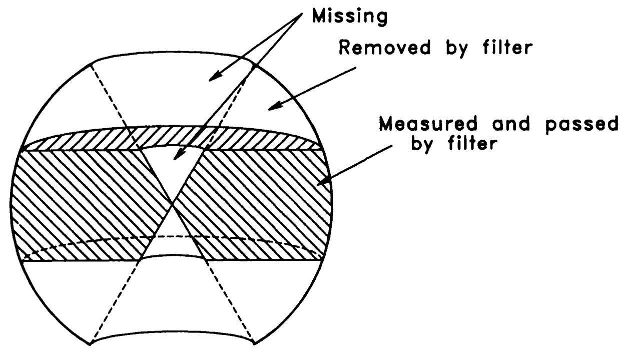

FIGURE 3
Illustration of the resulting region in the 3-D Fourier space after application of the ectomography algorithm.

quantitative one. Rather, they stress the potential resolution advantages of the RSH collimator and suggest that ectomography might be an effective algorithm to use with it in some clinical situations. Their contribution is a valuable first step in the clinical evaluation of this combination.

Among linear filtering algorithms, ectomography is a rather sensible one. Its effect in the Fourier domain is illustrated in Figure 3, adapted from Knutsson et al. (*12*). There is no attempt to fill in the missing cones; instead, the Fourier components far from the equatorial plane are suppressed by the filter. Within the pancake region that the algorithm attempts to restore, only a small notch is missing. The result is a system with very limited depth resolution, but with few artifacts and good in-plane resolution (*1*,*2*). We classify ectomography as classical tomography since object structures from a substantial range of depths contribute to the reconstruction of any plane of interest.

Ectomography is thus a less ambitious algorithm than many pursued in the limited-angle field. It remains content to produce classical tomograms with a modicum of depth discrimination rather than attempting to emulate CT and fill in the missing cones by use of questionable prior information. The images produced by Dale and Bone are an indication that this limited goal is more likely to prove clinically useful than the more ambitious (and probably unattainable) one of producing true computed tomograms from limited-angle data.

On the other hand, ectomography is not yet the ideal algorithm, even for this limited goal. It does not incorporate any prior knowledge, even the positivity constraint, and it does not yield images that are consistent with the measured data. Further development within the framework of classical tomography is certainly possible.

OUTLOOK FOR THE NINETIES

Algorithmic research per se, which has dominated the field of limited-angle tomography in the seventies and eighties, does not appear likely to produce great advances in the nineties. The most critical ingredient in image quality is the type and quality of the data collected. Beyond that, the kind of prior information incorporated is also very important, but the algorithm itself is far less important. Any two algorithms that intelligently use the same data and the same prior information will probably have very nearly the same objective clinical value.

A pressing need in this field, and indeed in all of medical imaging and image science, is a good way of quantifying image quality in terms of clinical efficacy. In spite of much research in this direction, we are still not at the point where precise quantitative assessments are possible. The images of Dale and Bone certainly suggest that their approach is clinically efficacious in comparison to planar imaging, but how are we to make that statement quantitative? Far more effort on objective, task-specific assessment of image quality in realistic clinical situations is needed, not only for comparing algorithms and imaging systems, but also for quantifying the value of different forms of prior knowledge. The value of positivity or support constraints or statistical prior knowledge can only be inferred indirectly with current methods of quality assessment.

Though it appears very unlikely that limited-angle *computed* tomography, in the sense in which we have defined it, will ever be possible, it does not follow that limited-angle methods have no clinical value. There are some clinical situations, such as the intensive care unit, where SPECT is simply impractical. Moreover, as pointed out by Dale and Bone, the limited-angle methods offer a potentially useful tradeoff between lateral and depth resolution, with better lateral resolution than SPECT combined with a rough depth discrimination absent in planar imaging. The challenge of the nineties is to enhance that capability by optimal data acquisition and effective use of prior information and to determine, objectively and quantitatively, when it is useful.

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REFERENCES

1. Dale S, Bone D. Tomography using a rotating slant hole collimator and a large number of projections. *J Nucl Med* 1990; 31:1675–1681.
2. Dale S, Bone D. Thallium-201 myocardial tomography using a rotating slant hole collimator and a large number of projections. *J Nucl Med* 1990; 31:1682–1688.
3. Hodes PJ, DeMoor J, Ernst R. Body-

section radiography: fundamentals. *Radiol Clin North Am* 1963; 1:229–244.
4. Vallebona A. A method of taking roentgenograms which makes it possible to eliminate shadows. *Fortsche. a.d. Geb. d. Roentgenstrahlen* 1933; 48:599–605.
5. Kuhl DE, Edwards RQ. Image separation radioisotope scanning. *Radiology* 1963; 80:653–662.
6. Anger HO. Tomographic gamma-ray scanner with simultaneous readout of several planes. Lawrence Radiation Laboratory Report UCRL-16899, 1966.
7. Patton JA, Brill AB, Erickson JJ, et al. A new approach to the mapping of three-dimensional radionuclide distributions. *J Nucl Med* 1969; 10:363.
8. Koral KF. Partial angle emission computed tomography. In: Williams LE, ed. *Nuclear medical physics, Volume II*. Boca Raton: CRC Press; 1987:155–175.
9. Rangayyan R, Dhawan AP, Gordon R. Algorithms for limited-view computed tomography: an annotated bibliography and a challenge. *Appl Opt* 1985; 24:4000–4012.
10. Davison ME. The ill-conditioned nature of the limited angle tomography problem. *SIAM J Applied Math* 1983; 43:428–448.
11. Edholm P, Granlund G, Knutsson H, Petersson C. Ectomography–a new radiographic method for reproducing a selected slice of varying thickness. *Acta Radiol Diag* 1980; 21:433–442.
12. Knutsson HE, Edholm P, Granlund GH, Petersson CU. Ectomography—a new radiographic reconstruction method—I. Theory and error estimates. *IEEE Trans Biomed Eng* 1980; 27:640–648.

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2. **A 30-year-old married woman had a 10-year history of ulcerative colitis. Periodic barium enemas were performed to monitor her disease and to look for the presence of maligancy. Her most recent barium enema was judged to be suboptimal and the examination was repeated 3 weeks later. The patient was subsequently found to be pregnant (2 weeks at the time of the first barium enema). The radiation dose to the embryo from each procedure was 3.1 rads (0.031 Gy). Which *one* of the following statements is correct?**

A. The likelihood of a radiation-induced congenital abnormality in the child is negligible.
B. There is a high risk of mental retardation in the child.
C. There is a high risk only for skeletal anomalies in the child.
D. The likelihood of congenital abnormalities is dependent on the total dose of 6.2 rads.
E. Therapeutic abortion should be advised.

3. **Medical radiation doses to the public average about 100 mrems/year/person. Which *one* of the following statements is correct concerning the need to reduce medical doses even further?**

A. There is no need to reduce doses further because the doses are only 2% of the legal occupational dose limit.
B. Doses should be reduced further, because nonstochastic effects can be seen after many years of exposure at these dose levels.
C. The U.S. Congress mandated a federal effort to reduce medical doses in the Atomic Energy Act of 1954.
D. The collective dose is quite high and of concern on a population-wide basis.
E. The only real concern is dose from nuclear medicine procedures, because of the internal deposition of the radioactive material.

SELF-STUDY TEST

Radiobiology and Radiation Protection

ANSWERS

ITEM 1: Hormesis
ANSWER **C**

The effects on humans of large radiation doses delivered at high dose-rates are relatively well known, i.e., the shape of the dose–response curve is well-defined. In contrast, the effects that might result from exposure to small doses of radiation in a protracted, low dose-rate pattern are not known with any degree of certainty. This lack of certainty regarding the effects of radiation at low doses and low dose-rates is largely due to the fact that the effects are likely to be identical to those caused by any number of other agents, such as toxic chemicals and chronic tobacco use. If a radiation effect is to be observed, it must occur with sufficient frequency in the irradiated population that this frequency of occurrence can be distinguished from the normal "background" incidence of the effect. In the absence of large-scale epidemiologic studies involving hundreds of thousands or even millions of individuals exposed to small doses of radiation above the background level, the derivation of a dose–response curve in the low-dose region requires extrapolation from the dose–response curve derived from high dose data. It is this extrapolation that introduces the uncertainty and controversy.

Radiation protection regulations must be written in a conservative manner, such that exposure to the doses permitted in the regulations does not lead to significant excess risk to the exposed person. Because the few data points that exist between the high-dose region of the dose–response curve and the zero-dose axis are scattered and do not have an exact geometrical relationship to each other, a mathematical model must be assumed and employed to complete the dose–response curve in this region. The model currently enjoying favor among national and international scientific advisory bodies is the linear–quadratic model, in which the lowest-dose region behaves according to a linear model of shallow slope and the remainder of the low-dose region behaves according to a quadratic model. This model agrees reasonably well with the sparse experimental and epidemiologic data and with the increased body of radiobiologic data that shows that the ability of a living system to repair low-dose damage may be greater than previously thought. The linear model is preferred by regulatory agencies, such as the U.S. Nuclear Regulatory Commission. This model connects a straight line from the bottom end of the high-dose response curve to the zero-dose/zero-effect intercept of the curve. It is considered to be suitably conservative by the regulatory community and by the overwhelming majority of the scientific community. A minority of scientists insists that a supralinear model fits the data just as well as the other two models. The supralinear model postulates that the effects of radiation per rem at low doses are more severe than at high doses, so that the dose–response curve is elevated above the

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