The Buildup Factor: Effect of Scatter on Absolute Volume Determination

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We have developed a new method for generating attenuation-corrected images for use in absolute volume and activity measurements. The technique relies on the use of a set of measured buildup factors to correct for the effects of scatter inherent in the broad-beam conditions of clinical nuclear medicine and requires anterior and posterior count-rate measurements. The scatter correction requires that the well-known attenuation factor $e^{-\mu d}$ be replaced by $1 - (1 - e^{-\mu d})^{B(\infty)}$, where $B(\infty)$ is the buildup factor at infinite depth. The buildup factors for four different scintillation camera window settings and three different source sizes are reported. The method was validated by calculating phantom volumes and comparing the results to a previously reported technique which does not account for the scatter contribution by assuming $\mu = 0.15\ cm^{-1}$. The results showed that the buildup factor method provides less than 7.3% error for volume determinations at all investigated depths, window settings, and source sizes, whereas errors of 3.3–26.7% were found with the other technique.

J Nucl Med 26:390–394, 1985

The use of relatively wide window settings in clinical nuclear medicine imaging results in significant scattered radiation. Count-based radionuclide methods for measuring absolute ventricular volumes are complicated by the need to correct for tissue scatter which is present under such broad-beam conditions. In order to develop a general method for scatter correction, a buildup factor was introduced (*1–4*), and successfully applied to the measurement of ventricular volumes (*5*) and general absolute activity measurements (*3,6*).

In the current study we have further investigated the dependence of absolute volume measurements on the buildup factor as a function of gamma camera window setting and depth. Preliminary findings on source size dependence are also reported. A new method for attenuation correction in planar imaging has been developed which may also have application in single photon emission computerized tomography.

MATERIALS AND METHODS

A scintillation camera fitted with a low-energy, parallel-hole collimator and interfaced to a commercial nuclear medicine computer system was employed for these studies. Window settings of 15%, 20%, 25%, and 30% were used. A thin, circular, 10-cm-diam source was prepared containing 650 μCi (24 MBq) of technetium-99m (^{99m}Tc) pertechnetate. The source was counted in air and at multiple depths in a 30 × 30 × 20 cm phantom of tissue equivalent material (Mix D) for the four window settings and the buildup factor, B(d), determined. In this method, referred to as the depth-dependent buildup factor (DDBF), B(d) is calculated (*3*) according to:

$$B(d) = C/C_0 \times e^{\mu d} \quad (1)$$

where

C = count rate measured for source at depth d in phantom;

C_0 = count rate measured in air for same source-to-collimator distance;

μ = narrow beam linear attenuation coefficient;

d = source depth in cm.

A second method for determining the buildup-factor was compared to the first by plotting the transmission factor, TF (TF = C/C_0) as a function of depth d in the same phantom material. These data were analyzed by a nonlinear least-squares fitting routine using the function:

Received July 30, 1984; revision accepted Dec. 28, 1984.

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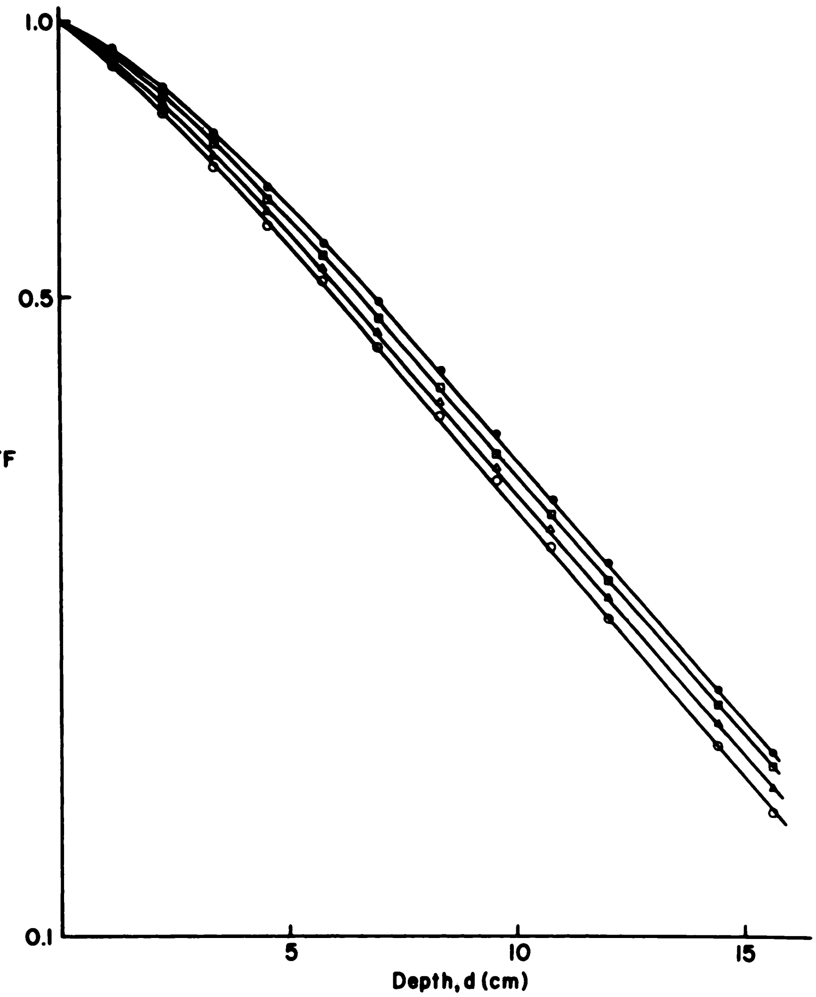

FIGURE 1
Transmission factor, TF, as function of depth obtained in phantom of tissue-equivalent material for four window settings. (O) 15%, (Δ) 20%, (□) 25%, (●) 30%

$$TF = 1 - (1 - e^{-kd})^n. \qquad (2)$$

At large depths, this reduces to $TF = ne^{-kd}$ where n is the buildup factor at infinite depth $B(\infty)$, and k is the narrow beam linear attenuation coefficient, μ. The parameters $B(\infty)$ and μ are determined by the nonlinear least squares algorithm. This technique will be referred to as the depth-independent buildup factor (DIBF) method.

A 150-ml cylindrical volume source of water containing 700 μCi (26 MBq) of [^{99m}Tc]pertechnetate was also prepared. The source thickness was 4 cm and the cross-sectional area was 37.4 cm^2. The source was positioned at various depths in the tissue-equivalent material for the four window settings. Anterior and posterior count rates were obtained and a 20 ml aliquot was counted in a petri dish at 10 cm from the face of the collimator. The count rates were determined using a semiautomated edge detection algorithm.* The volume was calculated for depths from 2 to 10 cm using both buildup factor methods. For comparison we also performed the same volume measurements using the method of Links et al. (*7*) which does not correct for the scatter contribution.

The volume determination is based upon the equation, volume = C_o/C_{alq} where C_o is the attenuation corrected source count rate and C_{alq} is the 20 ml aliquot count rate per ml. In order to determine C_o the buildup factor techniques require anterior (C_A) and posterior (C_P) count rates of the source or organ region of interest and a total phantom or patient thickness (T) measurement. The methods (DDBF, DIBF, and Links) for the measurement of C_o and the absolute volume are summarized in the Appendix.

In order to determine the dependence of μ on source size, we analyzed previously published data for 2 × 2 cm and 15 × 15 cm sources (*3*).

RESULTS

The measured transmission factor (TF) is plotted compared with depth d for all window settings in Fig. 1. These curves are not simple exponentials but have a sigmoid shape with an initial shoulder region followed by an exponential portion. The results of the nonlinear least-squares fit to these data are shown in Table 1. The narrow beam linear attenuation coefficient μ was un-

TABLE 1
Results of Curve Fit to Function $TF = 1 - (1 - e^{-\mu d})^{B(\infty)}$

Window setting (%)		Parameter μ	$B(\infty)$	Chi-square
15		0.137 ± 0.001	1.19 ± 0.01	2.6
20		0.139 ± 0.001	1.26 ± 0.01	3.9
25		0.139 ± 0.001	1.34 ± 0.01	5.6
30		0.138 ± 0.001	1.40 ± 0.01	5.1
	2 × 2 cm*	0.134 ± 0.001	1.37 ± 0.01	—
	15 × 15 cm*	0.137 ± 0.001	1.71 ± 0.01	—

* Results of previously published data (3).

affected by window setting and depth and found to be equal to 0.14 cm^{-1}. This is the correct value for μ for our tissue equivalent material. The results for the dependence of μ on source size are shown in Table 1 (30% window) and indicate that μ is also independent of source size. The depth independent buildup factor $B(\infty)$, however, is seen to vary as a function of both window setting and source size. The window width dependence was found to be in the form of a power curve $B(\infty) = a \times w^b$, where w is the window setting in %, a = 1.60 and b = 1.27. These results were obtained by pseudolinear regression analysis with a correlation coefficient of 0.99 and a standard error of the estimate equal to 0.06. It appears that the source size dependence is also a power function; however, it cannot be accurately determined since only three data points were obtained using two different camera-collimator systems.

The buildup factors (DDBF) calculated according to equation 1 (μ = 0.14 cm^{-1} was used due to the previous results) are shown in Fig. 2. These data were fitted by a nonlinear least-squares routine using a function of the form $1 + C(1 - e^{-md})$ and are shown in Table 2. This is equivalent to our previous functional form $A - Be^{-md}$ (5) but is more specific since at depth d = 0, B(d) = 1. The asymptotic value of the buildup factor (1 + C) is equal to the value obtained by fitting the TF data compared with depth (Fig. 2 and Table 1). This indicates that the n of Eq. (2) is indeed the buildup factor at infinite depth, $B(\infty)$.

The results of the absolute volume quantitation are shown in Table 3, assuming that the source thickness is known. The errors in the volume measurement for both buildup factor techniques (0–7.3%) are independent of depth and window setting. The buildup factor techniques used for volume quantitation were as follows: DDBF (A, Appendix); DIBF (B, Appendix). Using the Links technique (7) the errors (3.3–26.7%) are dependent upon both parameters, i.e., increasing with depth and window setting. Furthermore, the true depth d was used for the Links method. This certainly results in a best case measurement since ordinarily an independent depth measurement subject to its own uncertainty is required.

The average absolute errors for the DDBF, DIBF, and Links volume techniques (Table 3) were 5.1 ± 2.8 ml, 3.6 ± 3.0 ml, and 24.3 ± 9.1 ml, respectively, for all four window settings. Using a paired t-test, both buildup factor volume methods were statistically different than the Links method ($p < 0.001$) while the difference between the DDBF and DIBF techniques was only of borderline significance ($0.01 < p < 0.05$).

DISCUSSION

The relatively wide window settings used in clinical nuclear medicine shorten imaging time without a significant loss in image quality. However, if quantitative analysis is desired the effects of scatter must be taken into account.

The equation $TF = 1 - (1 - e^{-\mu d})^{B(\infty)}$ provides the key for deriving attenuation correction factors. As has

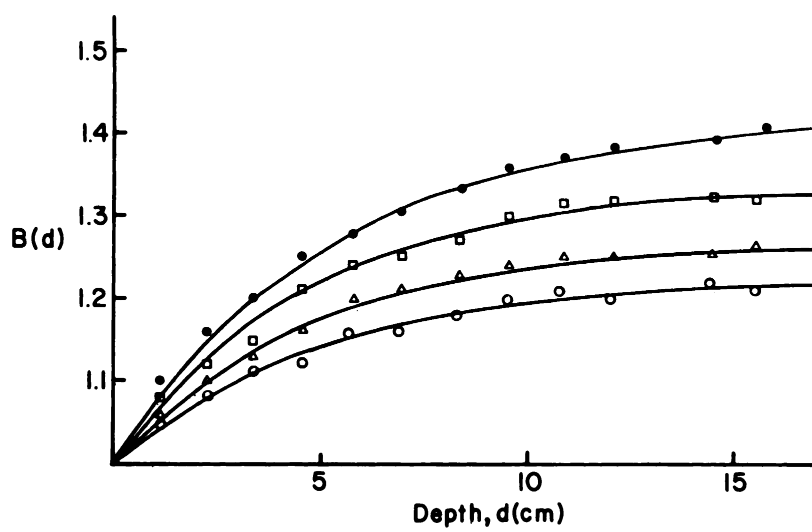

FIGURE 2
Buildup factor, B(d), as function of depth obtained in phantom of tissue-equivalent material for four window settings. (○) 15%, (△) 20%, (□) 25%, (●) 30%

TABLE 2
Results of Curve Fit to Function $B(d) = 1 + C(1 - e^{-md})$

Window setting (%)		Parameter C	m	Chi-square
15		0.23 ± 0.01	0.19 ± 0.01	1.8
20		0.27 ± 0.01	0.21 ± 0.01	2.5
25		0.34 ± 0.01	0.21 ± 0.01	4.0
30		0.42 ± 0.01	0.18 ± 0.01	8.7
	2 × 2 cm*	0.48 ± 0.01	0.16 ± 0.01	—
	15 × 15 cm*	0.72 ± 0.01	0.17 ± 0.01	—

* Results of previously published data (3).

been shown, μ is the narrow beam linear attenuation coefficient which is independent of window setting, source size, and depth. The buildup factor corrects for these sources of error. Since $B(\infty)$ probably varies with source size, given any source size, the value for $B(\infty)$ to be used in the transmission factor equation can be derived. This is applicable for attenuation correction in any quantitative nuclear medicine procedure. For example, in a study where absolute volume or activity measurement is desired, the source size (i.e., cross-sectional area) can be determined, the buildup factor derived and attenuation correction according to $TF = 1 - (1 - e^{-\mu d})^{B(\infty)}$ can be made.

TABLE 3
Comparison of Volume Techniques

Depth (cm)	Volume in ml (% error)* 15%† DDBF‡	15%† DIBF§	15%† L¶	20% DDBF	20% DIBF	20% L	25% DDBF	25% DIBF	25% L	30% DDBF	30% DIBF	30% L
2	152 (1.3)	145 (−3.3)	155 (3.3)	154 (2.7)	144 (−4.0)	156 (4.0)	146 (−2.7)	141 (−6.0)	155 (3.3)	149 (−0.7)	143 (−4.7)	161 (7.3)
3	157 (4.7)	150 (0.0)	164 (9.3)	156 (4.0)	149 (−0.7)	166 (10.7)	151 (0.7)	148 (−1.3)	168 (12.0)	154 (2.7)	148 (−1.3)	173 (15.3)
4	155 (3.3)	151 (0.7)	168 (12.0)	157 (4.7)	151 (0.7)	171 (14.0)	147 (−2.0)	147 (−2.0)	172 (14.7)	155 (3.3)	148 (−1.3)	179 (19.3)
5	157 (4.7)	149 (−0.7)	168 (12.0)	157 (4.7)	150 (0.0)	173 (15.3)	153 (2.0)	147 (−2.0)	176 (17.3)	155 (3.3)	147 (−2.0)	182 (21.3)
6	157 (4.7)	149 (−0.7)	170 (13.3)	158 (5.3)	148 (−1.3)	174 (16.0)	153 (2.0)	146 (−2.7)	178 (18.7)	154 (2.7)	146 (−2.7)	184 (22.7)
7	159 (6.0)	150 (0.0)	173 (15.3)	160 (6.7)	150 (0.0)	178 (18.7)	154 (2.7)	144 (−4.0)	179 (19.3)	154 (2.7)	144 (−4.0)	186 (24.0)
8	159 (6.0)	149 (−0.7)	173 (15.3)	160 (6.7)	149 (−0.7)	180 (20.0)	153 (2.0)	143 (−4.7)	180 (20.7)	154 (2.7)	145 (−3.3)	190 (26.7)
9	157 (4.7)	147 (−2.0)	172 (14.7)	160 (6.7)	148 (−1.3)	181 (20.7)	151 (0.7)	140 (−6.7)	178 (18.7)	153 (2.0)	144 (−4.0)	190 (26.7)
10	157 (4.7)	146 (−2.7)	172 (14.7)	159 (6.0)	147 (−2.0)	181 (20.7)	150 (0.0)	139 (−7.3)	179 (19.3)	152 (1.3)	142 (−5.3)	190 (26.7)

* Volume of source = 150 ml.
† Scintillation camera window setting.
‡ Depth-dependent buildup factor.
§ Depth-independent buildup factor.
¶ Links et al. (7).

The equation for the depth-dependent buildup factor, $1 + C(1 - e^{-md})$ would not be as ideally suited as a general scheme for attenuation correction since both C and m vary with window setting and source size (Table 2).

Either buildup factor method proposed in this study results in excellent volume determination at all depths and window settings. The DIBF method is easier to apply since only a single parameter, $B(\infty)$, is seen to vary with window setting and source size. Simpler computer codes may thus be used in accurate determinations of quantitative results compared with the DDBF method. The technique of Links et al. (C, Appendix) is seen to vary with all parameters, which is not surprising since the method does not account for scatter. The attenuation or transmission factor $e^{-\mu d}$ must be replaced by $1 - (1 - e^{-\mu d})^{B(\infty)}$.

In conclusion, a general scheme for attenuation correction has been developed. Once $B(\infty)$ as a function of source size is known the buildup factor can be calculated easily for any source size. This value can then be used in the transmission factor equation $1 - (1 - e^{-\mu d})^{B(\infty)}$ to obtain attenuation-corrected data, for use in absolute activity or volume determinations.

FOOTNOTE

* MUGE, Medical Data Systems.

APPENDIX

Methods of volume quantitation

$$\text{Volume} = C_o/C_{alq} \quad \text{(A)}$$

where C_o = attenuation-corrected source count rate

C_{alq} = 20 ml aliquot count rate per ml.

The following methods can be used to calculate C_o for use in equation A for volume determination:

A. Iterative buildup factor technique (3,5)

1. Measure anterior (C_A) and posterior (C_P) count rates

$$C_A = C_oB(d)e^{-\mu d}\,[\sinh(\mu x/2)/(\mu x/2)] \quad \text{(B)}$$

$$C_P = C_oB(T - d)e^{-\mu(T-d)}\,[\sinh(\mu x/2)/(\mu x/2)]$$

where B(d) is the DDBF expressed in the form $B(d) = [1 + C(1 - e^{-md})]$, C_o is the attenuation-corrected source count rate, x is the thickness in cm of the source, d is the anterior depth in cm to the center of the source, and T is the total phantom (or patient) thickness. The term in brackets is the source self-attenuation correction.

Equations B are solved iteratively for C_o

2. Measure anterior (C_A) count rate and anterior depth d

$$C_A = C_o[1 + C(1 - e^{-md})]e^{-\mu d}\,[\sinh(\mu x/2)/(\mu x/2)]$$

Therefore,

$$C_o = \frac{C_A}{[1 + C(1 - e^{-md})] \times e^{-\mu d} \times [\sinh(\mu x/2)/(\mu x/2)]} \quad \text{(C)}$$

B. Depth-independent buildup factor (DIBF) technique

1. Measure anterior (C_A) and posterior (C_P) count rates

$$C_A = C_o[1 - (1 - e^{-\mu d})^{B(\infty)}][\sinh(\mu x/2)/(\mu x/2)] \quad \text{(D)}$$

$$C_P = C_o[1 - (1 - e^{-\mu(T-d)})^{B(\infty)}][\sinh(\mu x/2)/(\mu x/2)]$$

Equations D can be combined as follows:

$$C_A/C_P = \frac{1 - (1 - e^{-\mu d})^{B(\infty)}}{1 - (1 - e^{-\mu(T-d)})^{B(\infty)}} \quad \text{(D')}$$

Equations D′ can be solved numerically for d by varying the value of d under the constraint $0 \leq d \leq T$.

Either of equations D can be used to calculate C_o.

2. Measure anterior (C_A) count rate and anterior depth d

$$C_A = C_o[1 - (1 - e^{-\mu d})^{B(\infty)}][\sinh(\mu x/2)/(\mu x/2)]$$

Therefore,

$$C_o = \frac{C_A}{[1 - (1 - e^{-\mu d})^{B(\infty)}] \times [\sinh(\mu x/2)/(\mu x/2)]} \quad \text{(E)}$$

C. Links et al. (7)

Measure anterior (C_A) count rate and anterior depth d

$$C_A = C_o\,e^{-\mu d}[\sinh(\mu x/2)/(\mu x/2)]$$

$$C_o = \frac{C_A}{e^{-\mu d} \times [\sinh(\mu x/2)/(\mu x/2)]} \quad \text{(F)}$$

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