Functional or Parametric Images

In Nuclear Medicine most images are, in essence, functional rather than structural, and for that reason the term "functional image" is not sufficiently precise for our purpose. Parametric, on the other hand, assumes that the data can be described in a closed equation defined by a restricted number of parameters. As an example, pulmonary washout can be represented by the equation $Ae^{-\lambda t}$. The parameters A and λ are sufficient to describe the entire kinetic behavior of the gas in the alveoli: A is a function of the ventilated volume and the equilibrium concentration of the tracer, λ of the fractional ventilation rate. If for each picture element (pixel), λ and A are determined from the observed count rate densities at different times during the washout, two parametric images are obtained (*1,2*).

Although the term parametric is frequently used, it has given the subject a luster of impracticality. To many, parametric images seem to be the product of academic, ludeic spirits, found particularly among physicists, but of little practical and clinical importance. Even mathematically, the term is not totally satisfactory. In (so-called) factorial analysis, the data are assumed to be described by the sum of a restricted number of primordial curves (eigenvectors). The parametric images are in this case formed from the coefficients (loading factors or factorials), which express the relative contribution of each of the eigenvectors to the total description (*3*). Those factors can hardly be described as parameters in the strict sense.

Parameter has also been used loosely to mean "a measure revealing one aspect of," as in: "The ejection fraction is a valuable parameter of ventricular function." In that improper sense, parametric and functional seem synonymous, since one assumes that the mapped parameter (say λ in an example given earlier) is related to, or descriptive of, a known physiological phenomenon under study (e.g., the fractional alveolar ventilation rate). This appealing interpretation, however, is not sufficient to define all the functional or parametric images that have come our way, and the absence of a clear understanding of the heterogeneity of this particular form of image has, in my view, hampered the development of the method as much as its acceptance.

It is preferable to think of functional or parametric images as particular forms of image processing that share one common characteristic: the procedure involves feature enhancement or information extraction of the original dynamic data and of necessity leads to concomitant information loss. Generally, the success of the operation depends on the value of the extracted features and the precision by which it can be computed.

Prejudice leads me to accept an implicit hierarchy in the types of features that can be extracted: (a) diagnostic, (b) physiological (true functional), (c) mathematical (true parametric), and (d) descriptive. An example of an application in the fourth, and perhaps the least attractive, class is one in which the first transit of a tracer was described by a "parametric" image representing the time at which the count rate density in each pixel reached its maximum (*4*). As a form of data reduction, this instance is relatively extreme, since a single time is used to define curves with varying amplitudes, some of which are actually bimodal (have two maxima). The intention was clearly not mathematical, nor was it assumed that the time had any particular diagnostic value *per se*. The parametric image, however, was used to determine a sampling region over the lungs and to determine those regions where a bolus was detected both in the pre- and the postpulmonary phase of the first transit. This approach is characterized by the fact that the original dynamic data cannot be reconstructed from the parametric images.

A more mathematical form of information extraction is the so-called phase analysis. Under certain conditions a mathematical function f(t) can be described by a sum of sinusoidal functions:

$$f(t) = a_0 + a_1 \cos(\omega t + ph_1) + a_2 \cos(2\omega t + ph_2) + a_3 \ldots.$$

The terms of this sum are, respectively, the zero harmonic (a_0), the first harmonic [$a_1 \cos(\omega t + ph_1)$], and so on. If by chance the higher terms do not contribute very much to the description of f(t), then f(t) is well approximated by the zero harmonic a_0 (which is its average value divided by two) and the first harmonic, which represents cyclic changes. The coefficient a_1 represents the size (amplitude) of the cyclic changes; the term ph_1 (for phase) represents the timing of this change. The "first harmonic" analysis has been applied mainly in the analysis of equilibrium-gated nuclear angiocardiography (*5–7*). The mathematics are correct, and the description of the data is perfect, to the extent that the terms $a_3, a_4, \ldots$ are indeed negligible. The relationship between a_1 and particular physiological values (i.e., the stroke volume) is less certain. There is no precise agreement, however, between the value of ph_1 and the timing of any single systolic event, unless the ventricular curve happens to be perfectly sinusoidal, in which case all the systolic event times are symmetric to each other.

The most attractive aspect of the "first harmonic" analysis has been the feature extraction. It is much easier to detect asynergy of the ventricle from a heterogeneity of phases in the left ventricular region than by watching moving displays of the original data.

The important mathematical aspect is that the original data can be reconstructed (or a fair approximation made) if one has an image of a_0, a_1, and ph_1. The same holds true for factorial images and for the parametric images describing pulmonary ventilation: A and λ define the original data set completely but again only to the extent that the model is true. For the ventilation, however, the striking aspect is the precise relation between a parameter (λ) and a well-defined physiological measure $\dot{V}/V$ (the fractional ventilation rate).

The same correspondence is assumed in the regional stroke volume images and ejection shell images (*8*). The stroke-volume image is the difference between the end-diastolic and end-systolic count rate densities on a point-by-point basis. If one assumes a constant concentration, position, absorption, and counting efficiency, this difference is proportional to differences in regional volumes at diastole and systole, i.e., regional contraction. The only weakness in the assumption is the one of constant position. The stroke-volume image, therefore, reflects local contraction *and* local motion. This type of parametric (or perhaps more adequately "functional") image is not mathematical, since the original data cannot be reconstructed from the images, but the immediate physiological relevance makes its use appealing.

Thus, the next step is illustrated in the article in this issue by Bacharach and his colleagues—it is the addition of diagnostic criteria to the information extraction (*9*). The distribution of phase values can be used to judge visually the synchronism of ventricular motion. In addition, Bacharach et al. compute a "reflected area" parameter from the distribution of phases, whose magnitude is related to the probability of having well-motion abnormalities.

If the production of parametric images implicitly connotes information loss (except in complete, truly mathematical parametric maps), how much additional information would be lost when the entire data set is reflected in a single parameter? The authors, however, point out first that the parameter includes spatial information, a truly original approach, and second, they indicate that the parameter is used to judge one clinical feature only.

Of course, the use of parametric images can be merely a sophisticated method for contrast enhancement (feature enhancement) to simplify the clinical analysis of dynamic data. The article by Stibolt et al. in this issue illustrates the point: the intention was to "show time-variant and spatial information in an easily assimilated form." The distinction between normal and abnormal remains subjective but is facilitated (*10*). Furthermore, there is no direct correspondence between distinct physiological phenomena and parameters. T_{MAX} can be influenced by blood clearance and by renal clearance, and the same is true for PI and $T_{1/2}$, although the present model did not reveal it. In general, clinicians are wary of that type of preprocessing that is perceived as prone to artifacts while the advantages (certain in my eyes) are perceived as marginal and easily compensated for by (clinical) experience.

A more attractive addition to simple enhancement is the introduction of normal ranges. We devised a pulmonary perfusion/ventilation ratio that did not show perfusion discrepancies until they reached a degree known to be indicative of pulmonary embolism (*11*). In essence that approach is similar to the one described here by Bacharach et al. and requires consideration of "error." Indeed, if we are to use parametric images, we have to realize that T_{MAX} will always exist, even if there is only noise; ratios will be computed, unless the denominator is zero; the clinician will be totally correct in insisting on reviewing the original data, unless the effects of random error are carefully under control.

But parametric images, well conceived, carefully computed, are here to stay, if only because the sheer volume of imaging now available will make exhaustive, but synoptic, examination impossible. As a class, however, they will be useful to the extent that:

1. They reveal an image feature of clinical importance.
2. They are relatively insensitive to error.
3. The mathematics are unequivocal.

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