A New Approach to the Smoothing of Dynamic Nuclear Medicine Data: Concise Communication

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A weighted three-dimensional digital filter that smooths data in both space and time has been developed for use with dynamic nuclear medicine studies. This smoothing algorithm allows a large improvement in signal-to-noise ratio without unacceptable degradation of spatial and temporal resolution. The initial results of using this smoothing algorithm suggest that it is superior to a standard nine-point smoothing function used on dynamic data. This is particularly encouraging since the parameters of the digital filter have not been optimized. The quality of the processed digital images is at least equivalent to that of analog images, and the digital images may be of more diagnostic value. The new algorithm also appears useful in preparing dynamic data for other manipulations such as the creation of parametric images or the extraction of quantitative measurements.

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The smoothing of noisy data is a well-recognized technique for the handling of data in the physical sciences and has been adapted for application to nuclear medicine (*1,2*). The most familiar forms of data smoothing are the nine-point smooth for images and the five-point smooth for curve data, and these are found on virtually all of the small dedicated computer systems used in nuclear medicine. Many other complicated smoothing algorithms, which are more properly called digital filters, have been developed for use with nuclear medicine images. For example, filters have been described that a) smooth an image so that it has a constant signal-to-noise ratio (*3*), b) smooth an image by acting as a band-pass filter (*4*), c) simultaneously smooth an image and provide edge enhancement qualities (*5*), and d) act as notch filters for rib erasure (*6*). Similar results may also be achieved by using Fourier techniques (*7*).

In the body of nuclear medicine literature associated with digital filtering, all of the techniques described are directed at improving a single or static gamma image (*3–15*). With the exception of the nine-point smooth, none of the filters described in the literature is widely used. Some reasons for this lack of acceptance might be a) lack of evidence that the proposed algorithm significantly improves diagnostic capability over analog data; b) lack of evidence that the proposed algorithm is significantly better than a nine-point smooth, or c) the fact that most computer displays are lacking in comparison with analog gamma-camera images—i.e., the display hardware generally available is not able to demonstrate fairly the capabilities of a particular filter. Of these possibilities, the last may be the most important because it has a direct effect on the others.

In contrast to most static studies, dynamic studies have an inherently high noise level because of low count statistics. Dynamic studies are therefore expected to benefit greatly from any digital filter appropriately designed to smooth such data. Furthermore, gamma-camera analog images of dynamic studies do not have the marked esthetic appeal that the static images generally have; thus it is more likely that one will appreciate improvement in the appearance of data from dynamic studies if an effective digital filter is used. This paper describes an ap-

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proach that has been used to design a digital filter for dynamic studies that gives a large improvement in the S/N ratio of nuclear medicine dynamic data, with an acceptable cost in terms of slightly decreased spatial and temporal resolution.

MATERIALS AND METHODS

Digital filters described in the literature have been designed to work on a single image, i.e., the filters are two-dimensional spatial filters (*3–15*). Dynamic studies, in contrast, are three-dimensional, consisting of two spatial dimensions and time. Thus, it would seem that a three-dimensional smoothing algorithm would be better matched to dynamic data than a two-dimensional algorithm.

To construct a prototype of a three-dimensional algorithm, consider a cube of points centered upon a point of interest. The cube chosen for investigation was a $5 \times 5 \times 5$ cube containing 125 points. This cube is formed from data points on the same data frame as the point of interest, plus similar points on the two preceding and two following data frames (Fig. 1A). Note that if all points were weighted uniformly, the improvement in the statistical S/N ratio would be approximately $\sqrt{125} = 11.18$. However, the loss of spatial and temporal resolution with such a filter would be unacceptable, and a weighted filter must be used.

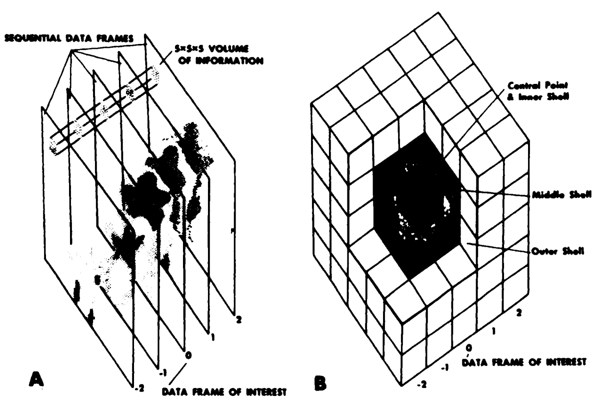

FIG. 1. Illustration of three-dimensional digital filter. (A) A $5 \times 5 \times 5$ volume of information is centered on each point of data frame 0 (illustrated here as early frame of brain-flow study). This volume of information is formed from corresponding 5×5 squares of data on 5 sequential data frames. Data in this cube of information are used to calculate weighted (smoothed) value for centered point. Sequence of frames is then shifted by one data frame until all frames in study have been filtered. (B) Each $5 \times 5 \times 5$ volume of information can be considered to be composed of three cube-shaped shells: an outer shell consisting of 98 points, a middle shell of 26 points, and an inner shell of one point, the central point or point of interest. Weighted (smoothed) value of central point is obtained by first summing values of all data elements of each shell; then by multiplying each shell sum by weighting factor; then by summing three weighted shell sums; and by finally dividing weighted sum by normalization factor. This operation is repeated for each point in data frame of interest.

To visualize a simple weighted filter, consider the $5 \times 5 \times 5$ point filter to be composed of three cube-shaped shells: a) an inner shell consisting of one point, the point of interest; b) a middle shell consisting of 26 points; and c) an outer shell consisting of 98 points (Fig. 1B). If the points in each shell are summed, the general expression for the digital filter under investigation is:

$$P = \frac{(W_I S_I + W_M S_M + W_O S_O)}{F}, \qquad (1)$$

where P = smoothed value of a point; S = shell sum; W = shell weight; F = normalization factor*; I = inner; M = middle; and O = outer. Obviously, three-dimensional filters of a much more complex nature can be readily be created.

Two dynamic digital filters were implemented in a Fortran IV program called SMVL, and used fairly routinely for several months. The filters operated on 64×64 dynamic (Byte mode) image matrices. The first filter (heavy SMVL) used a pure smoothing algorithm providing an increase in the S/N ratio of about 5.00 (Appendix A). The equation of this filter is:

$$P = (8S_I + S_M + S_O/7)/48, \qquad (2)$$

where P = smoothed data point value; S_I = sum of inner shell; S_M = sum of middle shell; and S_O = sum of outer shell.

The second algorithm (light SMVL) is a digital filter that lightly smooths the data and performs a mild edge enhancement. The improvement in the S/N ratio with this filter is about 1.95 (Appendix A). The equation for this filter is:

$$P = (37S_I + 2S_M - S_O/7)/75, \qquad (3)$$

where the symbols are the same as before. The equations for both filters were empirically derived†.

The major disadvantage in the use of either digital filter is the cost in computer time needed to process the data. For example, to process 20 frames of data from a brain flow would require about 430 sec. This time is too long for routine clinical applicability, especially if more than one pass of the algorithm is needed. For this reason the raw data are preprocessed so that the digital filter is used on acceptable data—i.e., data confined primarily to the organ of interest.

The preprocessing program called BKSP performs several functions. First, all points outside a circle 55 matrix elements in diameter, centered on a 64×64 element image matrix, are set to zero. Within this circle the program decides which elements contain acceptable data and which contain only noise. If an element is believed to contain noise, it is set to zero.

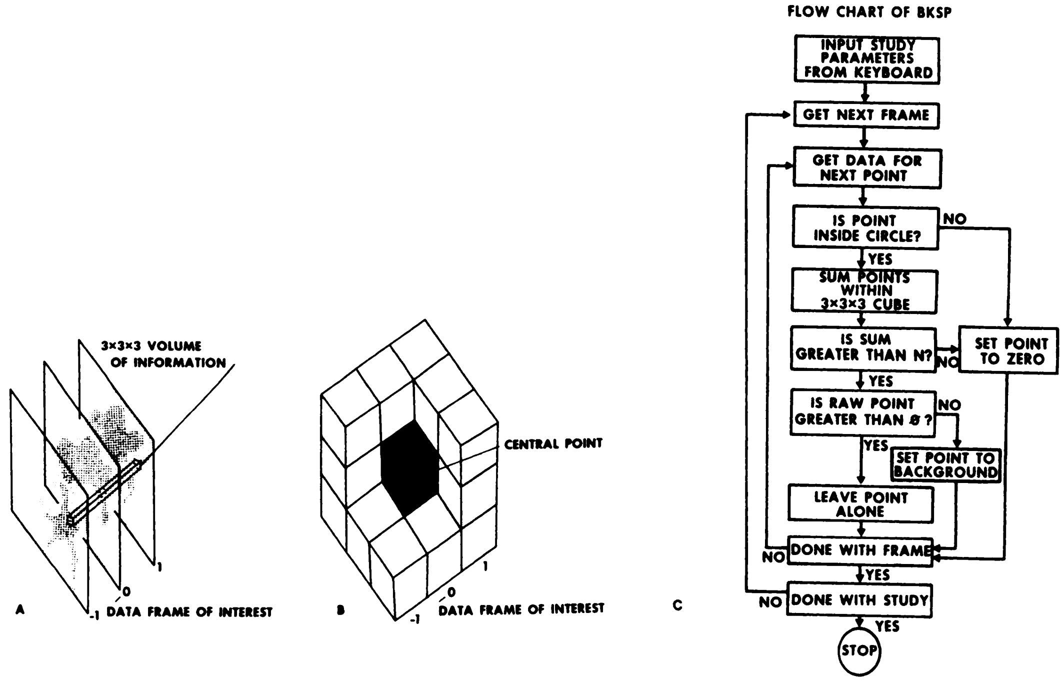

FIG. 2. Illustration of background-suppression program BKSP. (A) A 3 × 3 × 3 volume of information is formed from corresponding 3 × 3 squares of data on three sequential data frames. This volume of information is centered on each point of data frame of interest, and is used to decide whether or not centered point is acceptable data point. The sequence of frames is then shifted by one frame until all frames in study have been processed. (B) Enlargement of single 3 × 3 × 3 cube of information, demonstrating the relationship of central point on data frame of interest to remaining points in its cube. (C) Flow chart of logic behind BKSP. Purpose of BKSP is to leave data points within study region alone, while setting points outside this region to zero. Zero values within study region are set to background value. This process significantly decreases run time of SMVL by allowing SMVL to calculate smoothed value only for nonzero points. Note: *N* in this flowchart refers to test value discussed in text.

If an element is believed to contain valid data, it is left alone unless it has a value of zero. In this latter instance, the zero value is replaced with a background value‡ that is entered by the user at the start of program execution (Fig. 2C).

The test to see whether a data point is acceptable or not is similar in some respects to the smoothing algorithm itself. To test a matrix element, the program looks at a 3 × 3 × 3 cube of data centered on the point in question (Fig. 2A and B), sums the elements in this cube and then sees if this sum is larger than a test value‖ entered by the user at the start of program execution.

If the sum is larger than the test value, the point in question is considered to be a valid data point; if not, the point is considered to be invalid and is set to zero. Remarkably, BKSP works quite well for most data (Fig. 2C)§.

By starting with preprocessed data, the dynamic digital filter calculates new values only for nonzero data elements and thus has an improved run time. As an example, it now takes about 235 sec to process a 20-frame brain flow with BKSP (50 sec) and SMVL (185 sec) on our nuclear medicine computer¶. The saving in computer time is even more important when longer studies or multiple passes of SMVL are used.

RESULTS

An example of the comparative smoothing powers of various algorithms is shown in Fig. 3. The data in this picture show a portion of a renogram performed with 150 μCi of [^{131}I] Hippuran in a patient with bilaterally depressed renal function. The images are corresponding 30-sec images. For comparison purposes, the raw data (Fig. 3A) used for all smoothing operations were the output of the background-suppression program, BKSP. Figure 3B shows the results obtained when the standard nine-point smooth supplied by the manufacturer of our computer system was used twice on each image. Note that the data at this point are still extremely noisy. Figure 3C shows significant improvement in the S/N ratio when the heavy SMVL program is used once. Figure 3D shows an even greater reduction in the study noise with further improvement in renal visualization. The data in Fig. 3D were smoothed with a five-frame sliding smooth followed by the standard nine-point smooth used twice. The sliding smooth used in this instance replaced the frame of interest by the sum of five frames, centered on the frame of interest, divided by five. The results in Fig. 3E were obtained by applying the heavy SMVL program twice. Note that while the visualization of the kidneys is perhaps only slightly improved over that in Fig. 3D, the background noise has been almost totally eliminated.

An examination of the time/activity curves derived from the smoothed data is also of interest (Fig. 4). Figure 4A shows five curves taken from the left kidney: the raw-data curve; the nine-point smooth used twice; the heavy SMVL program used once; the five-frame sliding smooth, followed by the nine-point smooth used twice; and the heavy SMVL program used twice. The latter three curves are replotted in Fig. 4B for easier comparison.

Note that use of the nine-point smooth twice has not significantly reduced the noise in the resulting time/activity curve (Fig. 4A). SMVL used once or twice achieves a marked improvement in the appearance of the curve. Note that the peaks of both curves smoothed by SMVL appear to agree with respect to the raw-data curve (Fig. 4B). The time/activity curve generated from the five-frame sliding smooth followed by the nine-point smooth used twice also demonstrates a marked reduction in the amount of noise in the curve, but note that the peak of this curve is delayed by 1 min compared with all of the other curves (Fig. 4B). Thus, the improvement in the S/N ratio of the sliding smooth/nine-point smooth combination was achieved at the cost of some temporal distortion, whereas the weighted dynamic smoothing function achieved excellent improvement in the S/N ratio without significant temporal distortion.

The mild edge-enhancement property of the light SMVL program is shown in Fig. 5A, and is contrasted with the same data smoothed with the heavy SMVL program shown in Fig. 5B. Note that whereas the arteries appear sharper in Fig. 5A there is also more noise present in the remainder of the image as compared with Fig. 5B.

The critical question remaining is whether or not the computer-processed flow studies can add anything to the analog pictures. Figure 6A shows the anterior flow study obtained with 70-mm film at 2

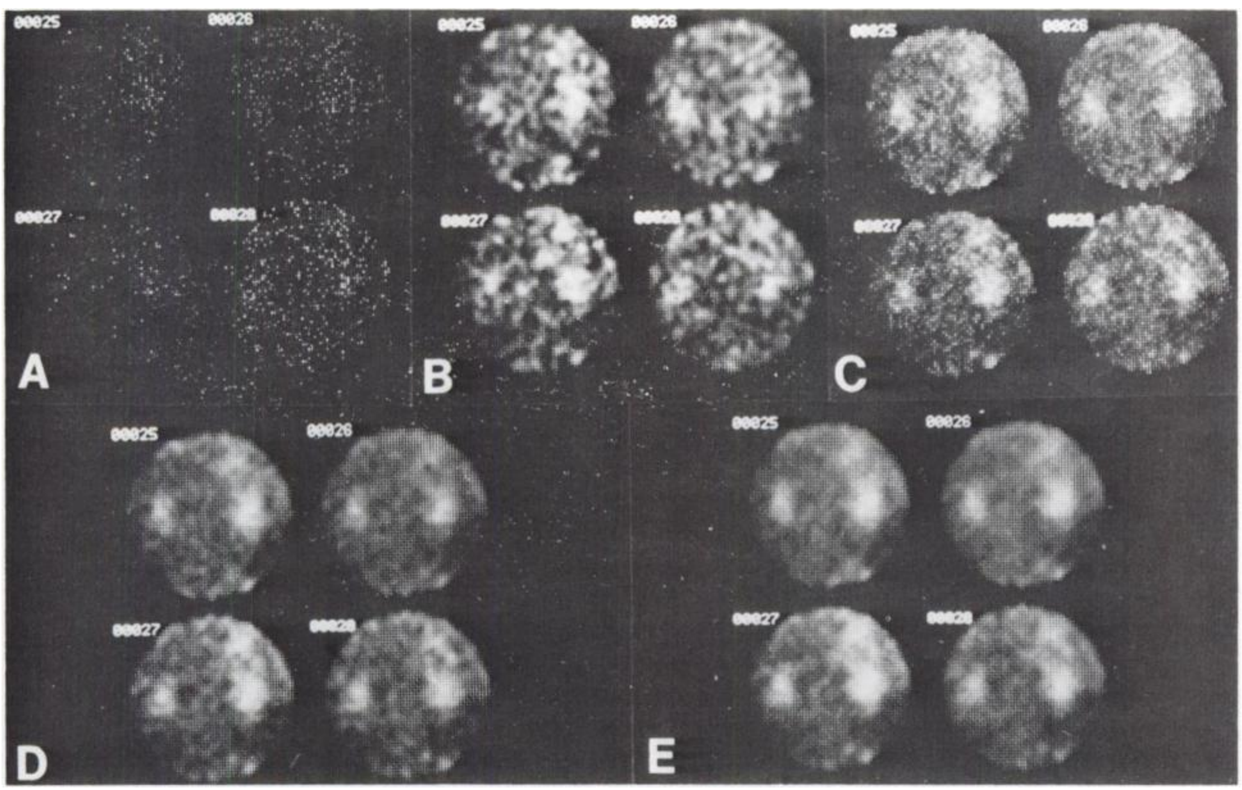

FIG. 3. Portion of renogram performed with 150 μCi of [^{131}I] hippuran in patient with bilaterally depressed renal function. Each segment of this figure consists of four sequential 30-sec images recorded from 12 to 14 min into the study. (A) Raw data, with kidneys practically unrecognizable in this low-count study. (B) Raw data have been smoothed twice with nine-point smooth. Visualization of kidneys is improved over raw data, but images remain quite noisy. (C) Raw data smoothed once by heavy SMVL; further improvement noted. (D) Raw data smoothed by a combination of a five-frame moving average (five-frame sliding smooth), followed by a nine-point smooth used twice. Marked improvement over raw data is noted. (E) Heavy SMVL used twice on raw data. Note that further reduction has been achieved in background noise, with perhaps slight improvement in visualization of kidneys in comparison with (D). Improvement in appearance of study processed with heavy SMVL used twice (E) is quite remarkable, considering appearance of raw data.

sec/frame from a patient with a left parietal stroke. The decreased flow over the left hemisphere in the arterial phase was noted initially. However, the subtle flip-flop (arrow) that is present in the analog pictures was not appreciated originally. The flip-flop is convincingly presented in the processed computer images (heavy SMVL $\times$ 1), at 1 sec/frame in Fig. 6B. The presence of a flip-flop phenomenon essentially limits the differential diagnosis to that of a stroke.

DISCUSSION

The initial clinical experience with this new algorithm is, unfortunately, anecdotal. It has been used on 100–150 brain-flow studies and on a smattering of gated cardiac studies and renal-flow studies. In virtually all instances, the image quality of the digitally processed data is equivalent to, or slightly better than, that of the analog data. Occasionally the processed data are diagnostically somewhat better than the analog images (Fig. 6B). The superiority of the processed data is particularly evident when the data are replayed in movie mode. Filtered studies replayed dynamically often show flow patterns and vascular structures seen with difficulty on dynamic replays of raw data.

The new dynamic digital filters are particularly useful for preprocessing data for further manipulation. Improvement in the Hippuran curve (Fig. 5A) is an example. Most of the experience with SMVL has come from use of the processed brain-flow data as an input to a program that generates parametric flow images. This processing has virtually eliminated random fluctuations as an explanation of flow asymmetries in the parametric images. In fact, the dynamic digital filters described were originally developed to improve the quality of brain-flow data for such further processing.

The results have presented the concept of a three-dimensional smoothing function for dynamic nuclear medicine studies. Furthermore, these results suggest that the dynamic digital filter may outperform simple nine-point smoothing or a sliding-smooth/nine-point-smooth combination. The improved appearance of dynamic studies processed by the dynamic filter may allow computer images to be the preferred display mode for dynamic nuclear medicine studies. It is also encouraging to note that the apparent advantages of using a three-dimensional digital filter were demonstrated with filters that were empirically derived. No attempt was made to optimize the filters. The only condition that these filters satisfied was that the resultant images looked good.

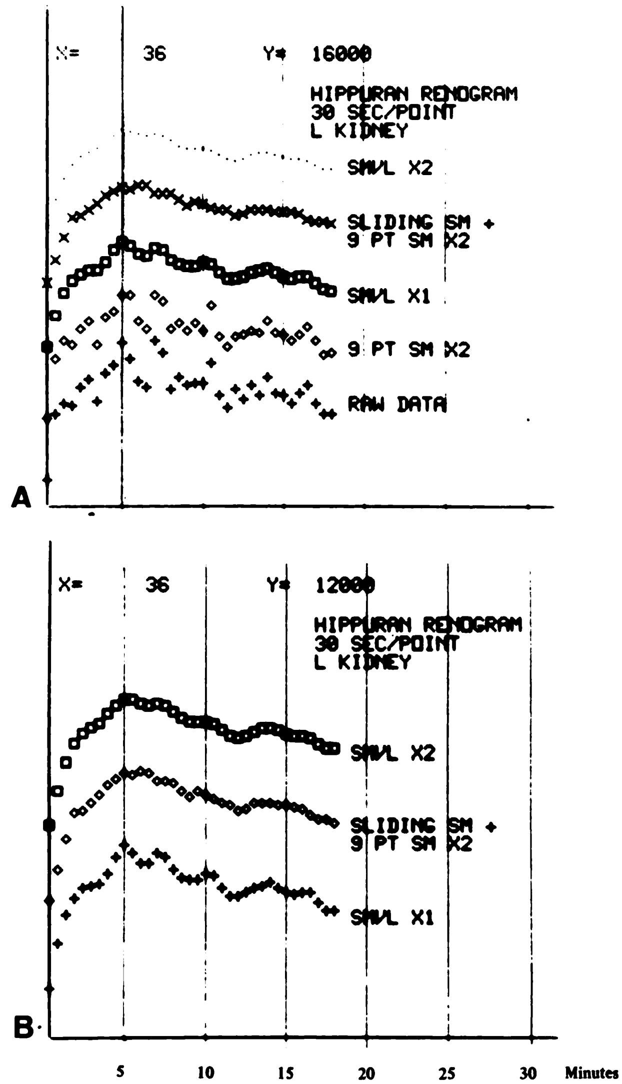

FIG. 4. Renogram curves derived from raw data and processed data of patient in Fig. 3. Five minutes elapse between each vertical bar. (A) Composite graph of renogram curves derived from raw data and data processed by four techniques used in Fig. 3. Note that renogram curve derived from data processed with nine-point smooth used twice is little improved over raw data, despite improvement in kidney visualization seen in Fig. 3B. This is because nine-point smooth filters data in space and not in time. All three curves filtered by algorithms that smooth in both space and time demonstrate marked reduction in noise. (B) Graph of upper three renogram curves from (A). Note that renogram curves derived from data processed once or twice with heavy SMVL peak at same time, and that this time appears to agree with raw-data curve. Close inspection of curve derived from data processed with combination of five-frame moving average (sliding smooth) and nine-point smooth reveals that peak of this curve occurs two points (1 min) later. Note also overall lack of distortion caused by using SMVL twice compared with SMVL used once. Only higher frequency transients have been further suppressed by using SMVL second time. Thus, three-dimensional digital filter, SMVL, appears to have insignificant degree of temporal distortion associated with its use.

SUMMARY

A weighted three-dimensional digital filter that smooths data in both space and time has been developed for use with dynamic nuclear medicine studies.

This smoothing algorithm allows a large improvement in signal-to-noise ratio without unacceptable degradation of spatial and temporal resolution. The initial results obtained with this smoothing algorithm suggest that it is superior to a standard nine-point smoothing function used on dynamic data. This is particularly encouraging since the parameters of the digital filter have not been optimized. The quality of the processed digital images is at least equivalent to that of analog images, and the digital images may be of more diagnostic value. The new algorithm also appears useful in preparing dynamic data for other manipulations such as the creation of parametric images or the extraction of quantitative measurements.

APPENDIX

The definition of the S/N ratio used in this paper is:

$$S/N = \frac{/\text{signal strenth}/}{\sqrt{\text{RMS noise}}} = \frac{/\text{signal strenth}/}{\sqrt{\text{signal variance}}}.$$

For nuclear medicine data and considering only random noise due to radioactive decay, this definitions becomes:

$$S/N = \frac{N}{\sqrt{N}} = \sqrt{N},$$

where N is the counts/pixel in the data (*17,18*)**.

To evaluate the effects of the dynamic digital fil-

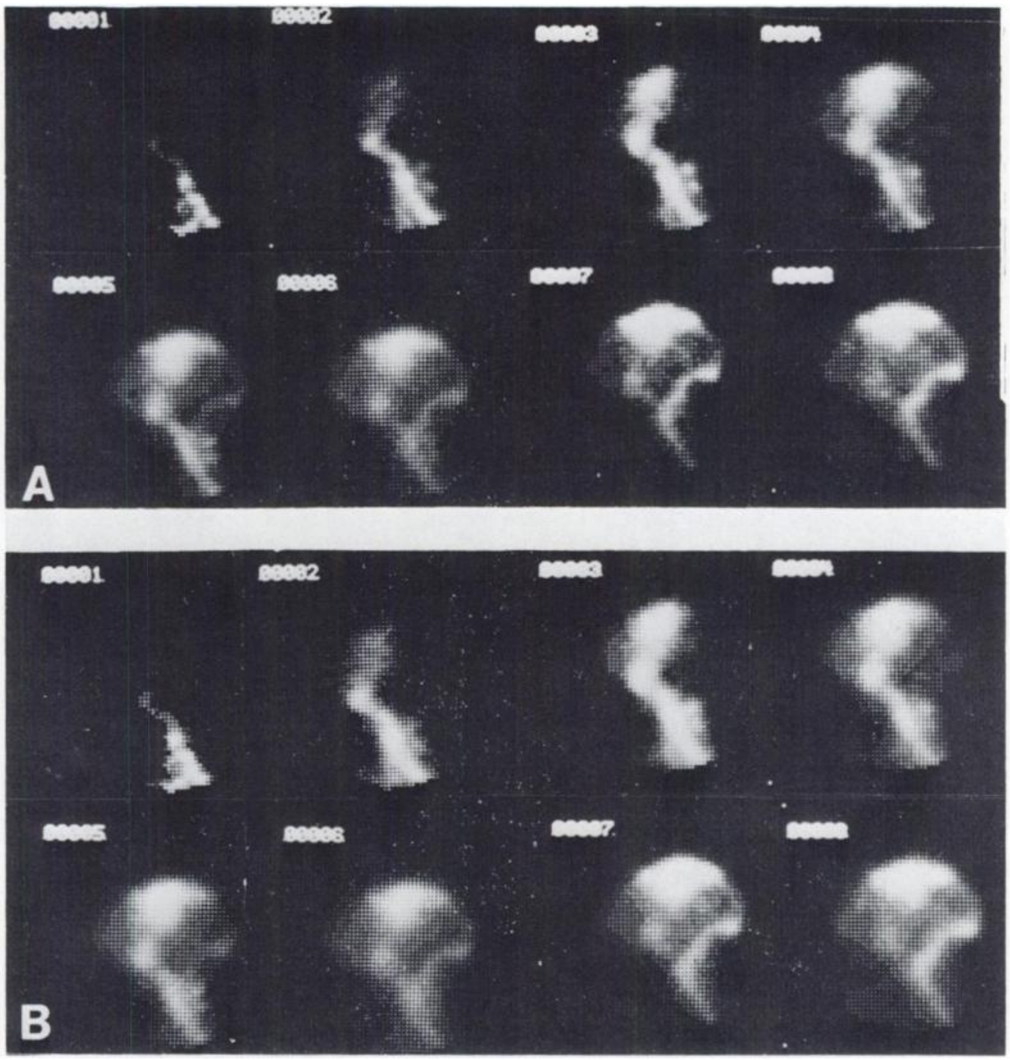

FIG. 5. Comparison of light and heavy SMVL. One-second scintiphotos of left lateral brain flow in patient with large parietal arteriovenous malformation. (A) Data processed with light SMVL used twice. (B) Data processed with heavy SMVL used once. Note that arterial and venous structures appear sharper in (A) than in (B). Close inspection of images reveals that there is slightly less noise in (B) than in (A). Because light SMVL was used twice, comptuer/time necessary to process (A) was almost twice as long as (B). Because both studies appear to be acceptable, choice of processing algorithm often comes down to slightly sharper images vs. longer processing time. Processing data with light SMVL once results in a study with obviously more noise than (A), but which is also obviously better than raw data.

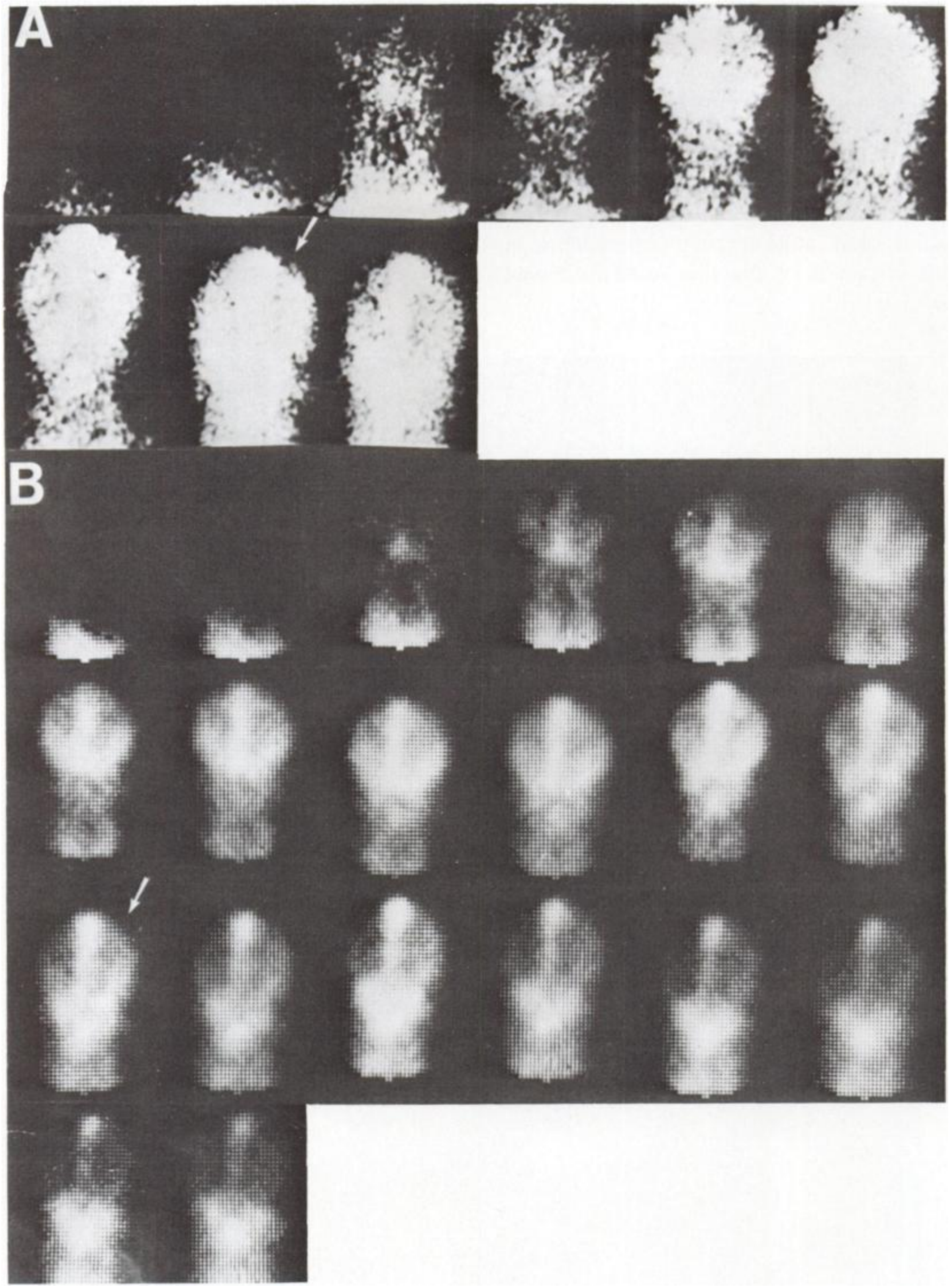

FIG. 6. Anterior brain-flow study in 32-year-old patient with left parietal stroke. Static images are positive. (A) Two-second sequential analog scintiphotos recorded on 70-mm film. Original analog images were overexposed, a problem with analog dynamic scintiphotography. Early flow deficit to left cerebral cortex is easily seen. Subtle development of flip-flop (beginning at arrow) was missed prospectively by several experienced observers. (B) One-second sequential scintiphotos processed using heavy SMVL once. Beginning development of flip-flop (arrow) is more easily seen, and convinced original observers of its existence.

ter, consider a dynamic study performed on a uniform flood-field source, with a resulting mean count N for each matrix element of the study. The signal strength of the raw data is N and the noise is $\sqrt{N}$. Thus, the S/N ratio of each pixel in this raw study is $\sqrt{N}$ as shown above.

From Eq. 1 in the text, the general expression for the dynamic digital filter used in this paper is

$$P = (W_I S_I + W_M S_M + W_O S_O)/F.$$

Note that when this filter is applied to the raw data of the flood-field study, the signal strength after using the filter will still be N (that is the function of the normalization factor, F). The noise in the data, however, will be changed.

To calculate the post-filter noise, one only needs to know that for a variable Y composed of weighted sums of independent variables X_i:

$$Y = \sum_{i=1}^{n} a_i X_i,$$

where a_i's are the weighting factors; the variance in Y is given by (*17,19*)

$$\mathrm{Var}\,(Y) = \sum_{i=1}^{n} (a_i)^2\,\mathrm{Var}\,(X_i).$$

Thus we see that the post-filter signal variance is

$$\mathrm{Var}\,(P) = (W_I/F)^2\,\mathrm{Var}\,(S_I) + (W_M/F)^2\,\mathrm{Var}\,(S_M) + (W_O/F)^2\,\mathrm{Var}\,S_O).$$

Because the shell sums S_I, S_M, and S_O are also linear sums of independent, Poisson-distributed data elements, it follows that

$$\mathrm{Var}\,(S_I) = \sum_{i=1}^{1} \mathrm{Var}\,(E_{Ii}),$$

$$\mathrm{Var}\,(S_M) = \sum_{i=1}^{26} \mathrm{Var}\,(E_{Mi}),$$

and

$$\mathrm{Var}\,(S_O) = \sum_{i=1}^{98} \mathrm{Var}\,(E_{Oi}),$$

where the E_i's are the matrix elements of the shells.

Because the expected value of any E_i in the above study is N, and because for Poisson-distributed variables, the variance equals the expected value:

$$\mathrm{Var}\,(S_I) = \sum_{i=1}^{1} N = N,$$

$$\mathrm{Var}\,(S_M) = \sum_{i=1}^{26} N = 26N,$$

and

$$\mathrm{Var}\,(S_O) = \sum_{i=1}^{98} N = 98N.$$

Thus the post-filter variance is

$$\mathrm{Var}\,(p) = \frac{(W_I^2 + 26W_M^2 + 98W_O^2)}{F^2}\,N.$$

From the definition of S/N ratio, and the knowledge that the post-filter signal strength is N and the post-filter variance is as just derived, the post-filter S/N ratio is

$$S/N = \frac{FN^{1/2}}{(W_I^2 + 26W_M^2 + 98W_O^2)^{1/2}}.$$

The improvement in the S/N ratio due to the digital filter is thus

$$\frac{F}{(W_I^2 + 26W_M^2 + 98W_O^2)^{1/2}}.$$

For the heavy SMVL filter, using the values for the shell weights and normalization factor from Eq. 2, the improvement in the S/N ratio is

$$\frac{48}{(8^2 + 26 + 98/7^2)^{1/2}} = 5.00.$$

For the light SMVL filter, using the values for the shell weights and normalization from Eq. 3, the improvement in the S/N ratio is

$$\frac{75}{(37^2 + 2^2 \cdot 26 + 98/7^2)^{1/2}} = 1.95.$$

If increased counts rather than a digital filter were used to improve the S/N ratio, it would require 25 times the number of counts for a S/N improvement of 5.00, and 3.80 times the number of counts for a S/N improvement of 1.95.

FOOTNOTES

* F is chosen for a dynamic study where all data elements equal 1; $P = (W_I S_I + W_M S_M + W_O S_O)/F = 1$; $P = (W_I + 26W_M + 98W_O)/F = 1$; $\therefore F = (W_I + 26W_M + 98W_O)$.

† The original filter, light SMVL, was based on knowledge of the five-point smooth found on the MED II (*16*). Although the processed dynamic studies were much improved over raw data, several passes of the filter were often required, and accordingly a heavier smoothing algorithm, heavy SMVL, was designed. The weights of heavy SMVL were chosen to give a S/N ratio of 5.00. Since brain-flow studies generally look good after one pass of heavy SMVL, further modifications have not been tried.

‡ Generally 1–3% of the maximum data-element count in the image is used as a background value, with 1 being the minimum value entered.

|| The test value entered is the background value previously used multiplied by a number between 1 and 27, generally 13. The output of BKSP must be examined for obvious artifacts before further processing continues. If these are recognized, adjustments for them (changes in the test value) are easily learned by technicians. When properly adjusted, BKSP produced little or no artifact. SMVL, itself, produces no artifact.

§ Data types that give BKSP difficulty are the following: (1) Very-low-count studies, i.e., studies with 10–12 counts/

data element as a maximum. Under these circumstances, SMVL must be used alone, and the longer run times tolerated. (2) Studies with very high and very low counts, e.g., heart-flow studies. In these studies, a minor degree of BKSP artifact is accepted. The artifact is a faint halo about the incoming bolus, which causes no difficulty in interpretation.

¶ Medical Data Systems Modumed, with Data General Nova 840 containing 32K of 800 manosec core memory and hardware multiply-divide.

** One author (*5*) has defined the S/N ratio as the square of the above definition where the subscripts and variables are as previously defined.

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