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Rapid, Rigorous Computation of Modulation Transfer Function on a Pocket Calculator

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This article describes a method for the rigorous calculation of modulation transfer function (MTF) of a collimated scintillation detector using a programmable pocket calculator. The calculation utilizes a normalized discrete Fourier transform (DFT) of the line spread function (LSF) including real and imaginary terms. A method is described for recording the LSF using an Anger camera and multichannel analyzer (MCA). The procedures for recording of the LSF and calculation of the MTF are therefore performed independently of a computer, allowing nuclear medicine laboratories without access to a computer to incorporate MTF studies into their quality-assurance programs. The results obtained using the pocket calculator were compared with those using a PDP-12 minicomputer. The pocket calculator was slightly more accurate, since it does not suffer from the round-off errors of the PDP-12. The time required to plot an 11-point MTF curve using the pocket calculator was 5 min, using the PDP-12 0.5 min.

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The two principal methods for experimental determination of modulation transfer function (MTF) of a collimated scintillation detector involve the use of either the Siemens sinusoidal sunburst phantom or a capillary-tube line source. The former method involves very simple computations, but is attended by a number of practical difficulties, including difficulty of construction of the sinusoidal phantom, difficulty of measuring C_t and C_0 (maximum and minimum count rates) at a single spatial frequency, detector response to scattered radiation within the phantom container, and requirement for performing measurements at a number of spatial frequencies (*1*). The line-source technique is easier to carry out experimentally and avoids the preceding problems of the sinusoidal sunburst phantom method, but measurements on a line source provide only the line-spread function (LSF) and the MTF must then be derived by calculating the normalized Fourier transform of the LSF. This is a complex calculation, but "if a computer is available to carry out the Fourier transform operation, this method for determining the MTF is preferred for its simplicity and accuracy" (*1*). This requirement for a computer has prevented the widespread use of the MTF in quality-assurance programs, particularly in smaller nuclear medicine laboratories.

This paper describes a method for the performance of MTF studies that is not dependent on the availability of a computer. The LSF is first measured using either a rectilinear scanner and scaler or an Anger camera and multichannel analyzer. The MTF is then calculated using a pocket calculator.

METHOD

1. Recording of LSF data. Accurate LSF measurements are crucial for MTF studies. The measure-

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TABLE 1. MTF PROGRAM LISTING USING THE HP-65 PROGRAMMABLE POCKET CALCULATOR

Memory address	Key entry	Machine code	Comments	Memory address	Key entry	Machine code	Comments
1	LBL	23	Beginning of subroutine 'B"	45	f	31	Rectangular to Polar Conversion of Complex MTF
2	B	12		46	R → P	01	
3	STO	33	Add R_x to R_3	47	R/S	84	Pause to read out|MTF|
4	+	61		48	.	83	Place 0.1 in R_x
5	3	03		49	1	01	
6	STO 7	3307	Store R_x in R_7	50	STO	33	Add R_x to R_2 (increment ν by 0.1)
7	2	02	2π	51	+	61	
8	g	35		52	2	02	
9	π	02		53	0	00	Zero R_3, R_4, R_5, R_8
10	x	71		54	STO 3	3303	
11	RCL 2	3402	Recall R_2 to R_x	55	STO 4	3304	
12	x	71	$2\pi \cdot R_2$	56	STO 5	3305	
13	RCL 8	3408	Recall R_8 to R_x	57	STO 8	3308	
14	x	71	$2\pi \cdot R_2 \cdot R_8$	58	LBL	23	Beginning of LSF and main calling routine
15	STO 6	3306	Store R_x in R_6	59	A	11	
16	g	35	Set trig mode to radians	60	2	02	1st point on LSF
17	RAD	42		61	B	12	Apply subroutine B
18	f	31	Take cosine of R_x	62	1	01	2nd point on LSF
19	COS	05		63	0	00	
20	x	71	$L(x_j,z) \cdot \cos 2\pi\nu_i\ (j-1)\Delta x$	64	B	12	Apply subroutine B
21	STO	33	Add R_x to R_4	65	3	03	3rd point on LSF
22	+	61		66	3	03	
23	4	04		67	B	12	Apply subroutine B
24	RCL 6	3406	Recall R_6 to R_x	68	7	07	4th point of LSF
25	f	31	Take sine of R_x	69	1	01	
26	SIN	04		70	B	12	Apply subroutine B
27	RCL 7	3407	Recall R_7 to R_x	71	1	01	5th point on LSF
28	x	71	$L(x_j,z) \cdot \sin 2\pi\nu_i\ (j-1)\Delta x$	72	0	00	
29	STO	33	Subtract R_x from R_5	73	6	06	
30	—	51		74	B	12	Apply subroutine B
31	5	05		75	8	08	6th point on LSF
32	RCL 1	3401	Recall R_1 to R_x	76	3	03	
33	STO	33	Add R_x to R_8	77	B	12	Apply subroutine B
34	+	61		78	4	04	7th point on LSF
35	8	08		79	4	04	
36	RTN	24	Return to main routine	80	B	12	Apply subroutine B
37	LBL	23	Beginning of subroutine "1"	81	1	01	8th point on LSF
38	1	01		82	5	05	
39	RCL 4	3404	Recall R_4 to R_x	83	B	12	Apply subroutine B
40	RCL 3	3403	Recall R_3 to R_x	84	5	05	9th point on LSF
41	÷	81	$R_4 \div R_3$	85	B	12	Apply subroutine B
42	RCL 5	3405	Recall R_5 to R_x	86	GTO	22	Apply subroutine 1
43	RCL 3	3403	Recall R_3 to R_x	87	1	01	
44	÷	81	$R_5 \div R_3$				

Register definitions: $R_1 = \Delta x$; $R_2 = \nu_i$; $R_3 = \Sigma L(x_j,z)$; $R_4 = \Sigma L(x_j,z) \cos 2\pi\nu_i\ (j-1)\Delta x$; $R_5 = \Sigma L(x_j,z) \sin 2\pi\nu_i\ (j-1)\Delta x$; $R_6 = 2\pi\nu_i\ (j-1)\Delta x$; $R_7 = L(x_j,z)$; $R_8 = (j-1)\Delta x$; R_9 (used by trig functions).

Label identification: Label A: Beginning of LSF storage and main calling routine; Label B: Beginning of MTF summation; Label 1: Beginning of MTF normalization.

ment of the LSF is relatively simple in the case of a rectilinear scanner (*2*). In the case of an Anger camera, a simple yet accurate way of measuring the LSF is to connect the camera to a multichannel analyzer (MCA), a relatively inexpensive device with many valuable uses even in the smaller nuclear medicine laboratory. We have used an Ortec Model 6220 MCA,* plugging the Y analog signal from our Picker Dynacamera 4/15,† which is normally fed to the persistence scope, into the AC input channel of the MCA. The "unblank" signal from the camera was plugged into the "gate" input of the MCA, so that only the flat portion of the Y analog signal would be analyzed by the MCA.

An approximation to a line source was made by placing a test tube containing 2 mCi of Tc-99m under a collimating slit between two lead bricks, 5-cm thick. The slit was 1-mm wide and 5-cm long. To record the intrinsic LSF the camera detector was positioned above the slit with the bricks touching the crystal surface and aligned parallel to the X axis of the detector. The LSF can also be recorded with

TABLE 2. USER INSTRUCTIONS FOR IMPLEMENTATING THE PROGRAM GIVEN IN TABLE 1

Step	Instructions	Input data/ units	Keys	Output data/ units
	To use example LSF:			
1	Load program			
2	Enter Δx (0.2 in this example)	0.2 cm	STO 1	
3	Compute MTF at $\nu = 0$ cycles/cm		A	MTF ($\nu = 0$)
4	Compute MTF at $\nu = 0.1$ cycles/cm		R/S	MTF ($\nu = 0.1$)
	Repeat 4 for each 0.1 frequency increment			MTF (ν_i)
	To use new LSF:			
1	Load program			
2	Index to LSF location		GTO A	
3	W/PRGM			
4	Enter $L(x_j,z)$ for x_1	$L(x_1,z)$		
5	Repeat 4 for all x_j	$L(x_j,z)$		
6	Terminate LSF		GTO 1	
7	RUN			
8	Enter Δx	Δx cm	STO 1	
9	Compute MTF at $\nu = 0$ cycles/cm		A	MTF ($\nu = 0$)
10	Compute MTF at $\nu = 0.1$ cycles/cm		R/S	MTF ($\nu = 0.1$)
11	Repeat 10 for each 0.1 frequency increment			MTF (ν_i)

a collimator in place, the distance from the slit to the collimator face being varied and scattering material being placed between slit and collimator.

The MCA was calibrated so that each channel was equivalent to a 2-mm displacement on the face of the camera's detector. Data were then accumulated in the MCA memory and read out both manually (using the movable cursor feature of the MCA) and automatically to punched paper tape. Manually obtained data were entered into the pocket calculator as described below and the data on punched tape were fed directly into our PDP-12 minicomputer, where a discrete fast Fourier transform (DFFT) program was used to verify the accuracy of the pocket calculator.

2. Calculation of MTF. The program described here has been written for the Hewlett-Packard HP-65‡ programmable pocket calculator. The HP-67, a more recent and powerful model, could also be used, but the program would need slight modification. The Texas Instruments SR-52‖ is a similar programmable pocket calculator for which an analogous program could be written.

The HP-65 is programmed using miniature magnetic cards containing up to 100 program instructions per card (224 in the case of the HP-67). Programs of greater length can be recorded on more than one card. To record programs or read them into the calculator, the cards are passed through a slot in the side of the calculator. Data are ordinarily stored in nine memory registers in the HP-65 (26 in the HP-67), which are also used for storage of intermediate results during the computation. The small number of data-storage registers is the major limitation of the HP-65. This drawback can be overcome to a certain extent, however, by recording data on program cards, either interdigitated with program instructions or on a separate card containing data only. Our MTF program stores both program and data on the same card.

MTF is calculated as follows.

$$\mathrm{MTF} = \frac{{}_{-\infty}\int^{\infty} L(x_j,z) \cdot e^{-2\pi i \nu x}\, dx}{{}_{-\infty}\int^{\infty} L(x,z)\, dx}$$

$$= \frac{\sum_{j=1}^{m} L(x_j,z) \cdot \{\cos 2\pi\nu x_j - i \sin 2\pi\nu x_j\}}{\sum_{j=1}^{m} L(x_j,z)}$$

$$= \frac{\sum_{j=1}^{m} L(x,z) \cdot \cos 2\pi\nu x_j}{\sum_{j=1}^{m} L(x_j,z)} - i \frac{\sum_{j=1}^{m} L(x_j,z) \cdot \sin 2\pi\nu x_j}{\sum_{j=1}^{m} L(x_j,z)}$$

$L(x_j,z)$ represents the value of the LSF at x_j; x_j is $(j - 1)\Delta x$ for $j = 1, 2, 3, \cdots, m$; m is the number of LSF samples to be input to the program; Δx is the constant sampling interval of LSF; ν is the frequency at which MTF is calculated.

The program for performing this calculation is shown in Table 1. Note that only 57 instructions are required for the body of the program, the remaining 43 locations on the card being available for storage of LSF data. In this example the LSF shown, together with subroutine instructions, occupy only 30 of the available 43 locations. The method for substituting new LSF data is shown in Table 2. Recording an LSF on the card takes about half a minute and computation of MTF at a single spatial frequency takes half a minute. The program features an automatic frequency increment to facilitate calculation of MTF over a range of frequencies. An MTF calculation from 0 to 1.0 cycles/cm in increments of 0.1 takes 5 min. It is a simple matter to modify the program to cover other frequency ranges or increments.

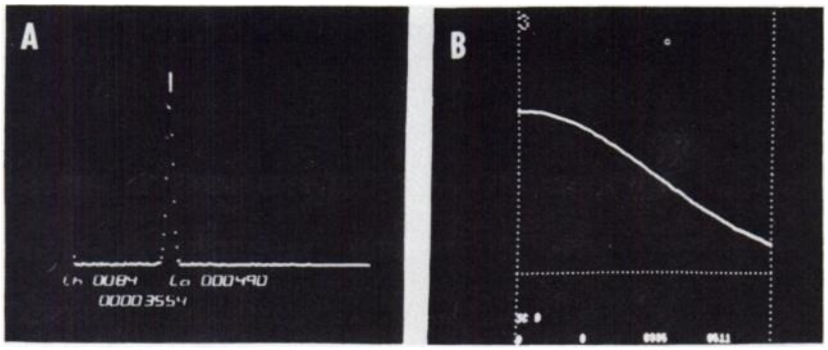

FIG. 1. (A) LSF recorded with Picker Dynacamera 4/15 and Ortec Model 6220 MCA as displayed on CRT display of PDP-12. (B) MTF curve obtained from (A) using the PDP-12 computer and DFFT program.

RESULTS

An LSF recorded using the Picker 4/15 camera and Ortec 6220 MCA is shown in Fig. 1A as displayed on the CRT display of the PDP-12. Figure 1B shows the MTF curve obtained on the PDP-12 using the DFFT program. Table 3 gives the actual LSF data and the MTF values from 0–1.0 cycles/cm as calculated by the HP-65 and the PDP-12.

DISCUSSION

No previous programming experience is required to write programs for the HP-65. A straightforward explanation of programming is given in the operator's manual and even the novice can write advanced programs within a couple of hours.

The computation of MTF using the HP-65 is a rigorous one, using a normalized discrete Fourier transform (DFT), including real and imaginary terms. The PDP-12 program uses a discrete fast Fourier transform (DFFT) (*3*), that is part of a previously described interactive curve-analysis program (*4*).

Comparison of MTF values calculated on the HP-65 with corresponding values obtained on the PDP-12 showed close correlation. Minor differences between the two are due to round-off errors in the PDP-12 calculations that are not present in the HP-65 calculations. The HP-65 uses floating-point arithmetic accurate to 10 significant decimal digits, whereas the PDP-12 uses fixed-point arithmetic and 12-bit (binary) words. The greater accuracy of the HP-65 is of little practical significance, however, since the plotted MTF curves using the two sets of results are virtually indistinguishable. The PDP-12 is significantly faster than the HP-65 (0.5 min compared with 5 min) but this is not a problem for the HP-65, since the computation may be performed only once a week.

The techniques described in this communication constitute a powerful tool for conducting parametric Anger-camera performance tests without on-line computer capability and at a small fraction of the cost of a dedicated minicomputer system. Applications of the programmable pocket calculator are, of course, not limited to the calculation of MTF. HP-65 programs have been written for a number of other commonly performed nuclear medicine computations (*5,6*).

TABLE 3. ACTUAL LSF DATA AND THE MTF VALUES FROM 0–1.0 CYCLES/CM AS CALCULATED BY THE HP-65 AND THE PDP-12

LSF measured with Picker Dynacamera 4/15 and Ortec Model 6220 MCA		MTF values calculated from accompanying LSF: Comparison of HP-65 programmable pocket calculator and PDP-12 computer		
Channel no. 0.2 cm/channel	L(x,z) counts/sec	Frequency	HP-65 results	PDP-12 results
1	2	0	1.000	1.000
2	10	0.1	0.983	0.982
3	33	0.2	0.935	0.932
4	71	0.3	0.859	0.854
5	106	0.4	0.763	0.756
6	83	0.5	0.656	0.646
7	44	0.6	0.546	0.543
8	15	0.7	0.441	0.433
9	5	0.8	0.346	0.340
		0.9	0.266	0.262
		1.0	0.201	0.199

ADDENDUM

Since this paper was submitted for publication, a related previous publication (Goldsmith WA, Nusynowitz ML: Determination of the modulation transfer function [MTF] using a programmable calculator. *Am J Roentgenol Radium Ther Nucl* 112: 806–811, 1971) was brought to our attention. Goldsmith and Nusynowitz used an approximation in the MTF computation which is valid only in the case of symmetric LSF. In practice, LSF measurements are almost always asymmetric, and for this reason we feel that the program described in the current paper represents a significant improvement in that it is valid for both symmetric and asymmetric LSFs.

FOOTNOTES

* Oak Ridge, Tenn.
† Cleveland, Ohio.
‡ Corvallis, Ore.
|| Richardson, Texas.

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6. Program numbers 04582A, 04583A, 04584A, 04585A, 04586A, and 04601, HP-65 Users' Library, 1000 N.E. Circle Blvd., Corvallis, Ore. 97330.

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