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Digital Low-Pass Filtering

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Random noise beyond the maximum signal frequency of the imaging system can be removed by low-pass filtering without compromising signal transmission.

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Visual recognition of the structure of an object is impaired by noise that is spectrally adjacent to the frequencies of interest. High-frequency noise, introduced by the sampling and quantitizing, that lies within two octaves of the signal is particularly effective in masking visual recognition (*1*). The method of blurring digital data by substituting for each element of a matrix the average of a subset of surrounding elements is still in use despite its infringement on signal transmission (*2*). The method of low-pass filtering detailed in this report transmits all signal (and noise) data up to the maximum frequency that the imaging system can transmit, but removes all noise beyond that point. Calculation of the filter requires knowledge of the transfer characteristics of the system.

MATERIALS AND METHODS

The modulation transfer function (MTF) was calculated for a Searle Radiographics Pho/Gamma HP scintillation camera with a high-sensitivity collimator. A ^{99m}Tc point source was imaged at a distance of 2 in. from the collimator in a scattering medium. Under these experimental conditions the MTF approaches zero at 0.6 cycles/cm. Radionuclide distributions are stored as 64 × 64 arrays of numbers on the Intertechnique Model-66 data acquisition system (Fairfield, N.J.). The element size is calibrated to be 0.4 × 0.4 cm. According to the sampling theorem, if a continuous function f(x) is sampled at least twice in any increment as large as the wavelength of the highest spatial frequency component in f(x), then f(x) can be exactly reconstructed from the discrete samples (*3*). Thus, the recording system is capable of storing information up to 1.25 cycles/cm. The noise band from 0.6 to 1.25 cycles/cm is removed by a spatial convolution of the digital image with the low-pass filter. On the assumption of radial symmetry, the processed image g(r), the unprocessed image f(r), and the filter function h(r) can be written as follows:

$$g(r) = 2\pi \int_0^\infty f(r')h(r' - r)r'\,dr'.$$

Fourier transformation reduces the integral equation to an algebraic equation:

$$G(\mu) = F(\mu)H(\mu).$$

The choice of $H(\mu)$ becomes immediately apparent from inspection of the MTF (Fig. 1). Since μ is a radial frequency component, the shape of the filter is that of a circular aperture in the frequency domain with a radius of 0.6 cycles/cm.

Recognizing that the Fourier transform of a circular aperture is a Bessel function of the first kind of order one, the low-pass filter h(r) is given by

$$h(r) = \frac{2\pi\mu_{max}\,J_1(2\pi r\mu_{max})}{2\pi r\mu_{max}},$$

where μ_{max} is 0.6 cycles/cm. The filter can be computed in a matter of minutes once μ_{max} is given. Note that $J_1(a)/a$ converges for $a = 0$. The filter is stored

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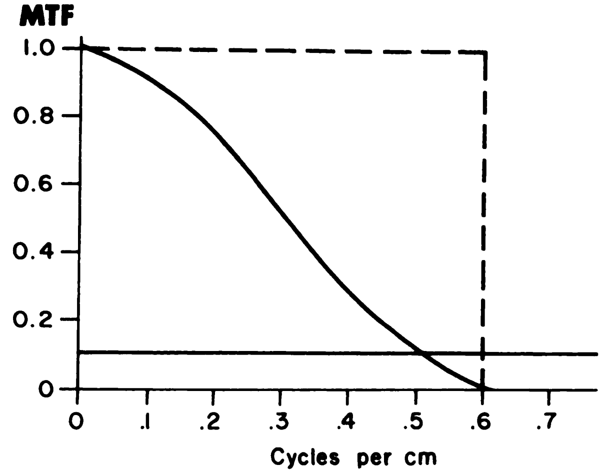

FIG. 1. Curved solid line represents isolated signal modulation, and straight solid line represents arbitrary magnitude of statistical noise. Since spatial frequency is radial, filter (dashed line) corresponds to circular aperture in spectral domain.

as a variable 3 × 3 to 25 × 25 element matrix. The filter function employed in this communication is of size 9 × 9. Since both image and filter are given as matrices, the convolution in practice is a matrix operation (*4*):

$$g_{ij} = \sum_{k,l=1}^{64} f_{k,l}\, h_{i-k,j-l}.$$

The spatial convolution is performed by the Multi-8 computer of the Intertechnique data acquisition system.

RESULTS

A liver phantom was imaged under conditions representative of the MTF of Fig. 1 for a total of 300,000 counts. Figure 2 shows the unfiltered digital display, along with successive low-pass filterings to 0.2, 0.4, 0.6, 0.8, and 1.0 cycles/cm. The optimum image, as predicted by the MTF, is obtained by low-pass filtering to 0.6 cycles/cm. A narrower spectral aperture results in loss of signal information. The image is sharpened as the upper frequency cutoff is extended from 0.2 to 0.4 and finally to 0.6 cycles/cm. No signal information can be gained beyond 0.6 cycles/cm, only random noise; and as the frequency range is extended stepwise to 0.8 and 1.0 cycles/cm, the image gradually acquires the noisy mottled appearance of the initial unprocessed display. The histograms of Fig. 3 are the profiles of the matrix row through the smallest and central void for the unfiltered image and for the image low-pass filtered to 0.6 cycles/cm. The interference of noise with recognition of spectrally adjacent structures is better exemplified by Fig. 4. The original images are on the left and the filtered ones on the right, displayed with the same intensity and identical photographic settings. In sequence from top to bottom, the images are those of a normal anterior brain (500,000 counts), abnormal posterior brain (500,000 counts), and abnormal thoracic spine (300,000 counts). The noise effectively masks the rib structure in the spinal view. In all views, pattern recognition is improved by low-pass filtering.

DISCUSSION

The ability to differentiate one area of a density pattern from another is a function of the sharpness of the border between the two. The high-frequency components of a spatial structure that are necessary for border definition are adjacent to noise beyond the maximum signal-transmittance capability of the system. We have shown that the masking associated with this high-frequency noise can be minimized by low-pass filtering. This method results in an improved signal-to-noise ratio without affecting signal content. The serial processing technique of this report compares favorably with the parallel processing

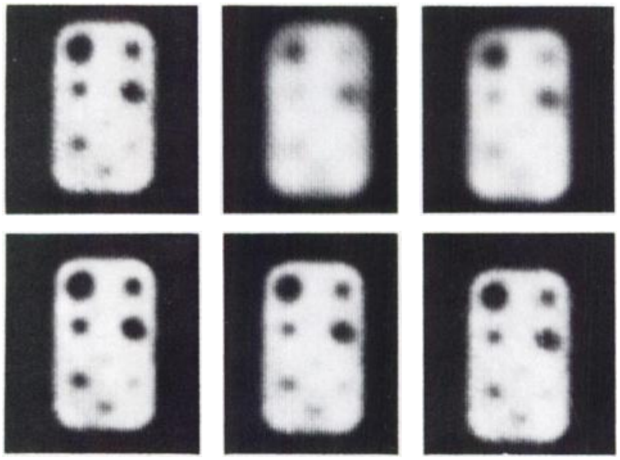

FIG. 2. From left to right, upper displays are unprocessed, low-pass filtered to 0.2 cycles/cm, and low-pass filtered to 0.4 cycles/cm, respectively. Lower displays are low-pass filtered to 0.6, 0.8, and 1.0 cycles/cm. Optimum image is obtained by low-pass filtering to 0.6 cycles/cm (lower left).

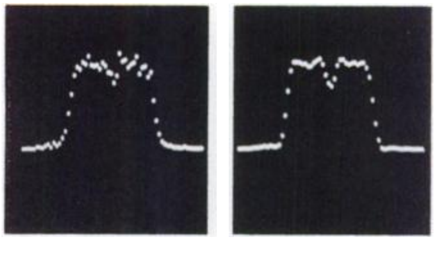

FIG. 3. (Left) Unprocessed matrix row through smallest void; (right) identical matrix row after optimal low-pass filtering to 0.6 cycles/cm.

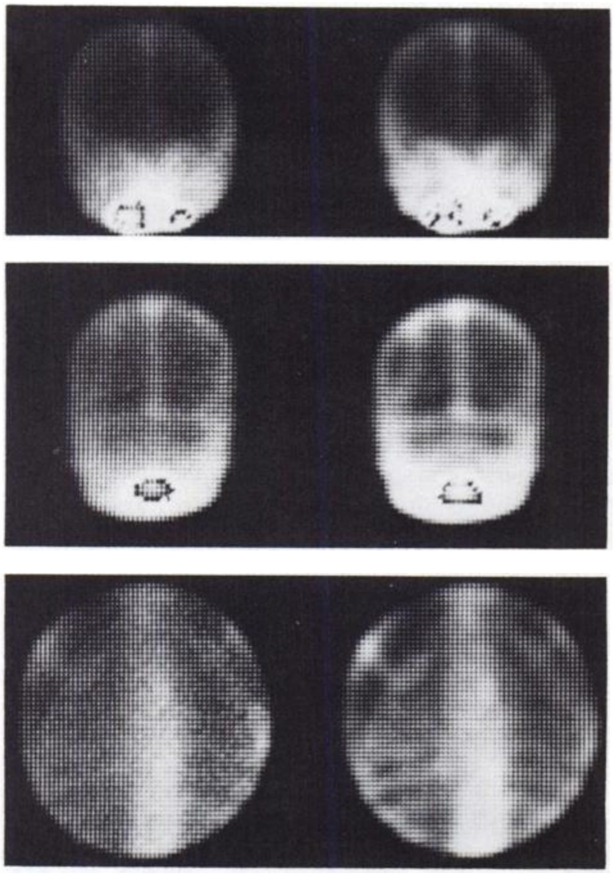

FIG. 4. Images of normal brain (top), abnormal brain (middle), and abnormal bone (bottom), with unprocessed displays on left and filtered displays on right.

method for low-pass filtering. The coherent optics technique utilizes a circular aperture in the frequency domain, with the point discontinuity at the border modified to a Gaussian convergence to zero (*5*). A point discontinuity in the frequency domain transforms to an oscillating function in the spatial domain which, in general, can either increase or decrease noise (*4*). The structures in Fig. 2 and 3 show no evidence of any oscillations. Oscillations will be found at the border of the camera field of view, but this, along with some inherent edge effect, should not prove troublesome to an experienced observer. The filtering operation requires approximately 40 sec, an effort well rewarded by the improvement in pattern recognition.

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INTERNATIONAL SYMPOSIUM ON COMPUTER-ASSISTED TOMOGRAPHY

An international symposium on "Computer-Assisted Tomography in Nontumoral Diseases of the Brain, Spinal Cord, and Eye," sponsored by the National Institute of Neurological and Communicative Disorders and Stroke, is announced. The meeting will be held at the Clinical Center on the campus of the National Institutes of Health, Bethesda, Maryland, on October 12–15, 1976, under the chairmanship of Giovanni Di Chiro, M.D.

The topics will include the physics, technologies (various devices and modalities), and the clinical (morphologic and functional) aspects of transmission and emission computer-assisted tomography of the brain, spinal cord, and eye, with emphasis on nontumoral diseases.

Scientific and technical exhibits are planned.

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