ANGER CAMERA DEADTIME

In a paper by Adams and Zimmerman (*1*) the statistical nature of the decay process has not been taken into account. Therefore, the deadtime of the system appears to depend on the input rate. The following reasoning makes this an unnecessary statement.

The following notation will be used: n is true input rate, r represents measured input rate, $\Delta r = r - n$ is error of the measurement, and $\epsilon = \Delta r/n$ represents relative error of the measurement.

From the fact that the decay process is a Poisson process, the following frequency distribution of time intervals between p decay events can be derived (T mean interval) (*2*):

$$\frac{g(p,t)}{n} = \frac{p^p}{(p-1)!}(nt)^{p-1}e^{-p(nt)} \qquad (1)$$

For $p = 1$, i.e., for every two adjacent events the formula reduces to

$$\frac{g(t)}{n} = e^{-nt}; \int_0^\infty \frac{g(t)}{n}\,dt = 1. \qquad (1a)$$

If such a signal enters a system with deadtime τ, the following amount of events is lost:

$$\epsilon = -\int_0^\tau \frac{g(t)}{n}\,dt = -(1 - e^{-n\tau}). \qquad (2)$$

Therefore

$$r = n(1 + \epsilon)$$
$$r = ne^{-n\tau} \qquad (3)$$
$$n = re^{n\tau}. \qquad (3a)$$

A first order approximation of the exponential function leads to

$$r \approx n(1 - n\tau).$$

This might be the link to the relations given in the paper by Adams and Zimmerman.

Now the deadtime may be calculated using Eq. 3a in combination with the two-source measurement described by

$$n_{12} + n_b = n_1 + n_2 \qquad (4)$$

$$r_{12}e^{n_{12}\tau} + r_b e^{n_b\tau} = r_1 e^{n_1\tau} + r_2 e^{n_2\tau}. \qquad (5)$$

This equation must be solved using numerical methods on a digital computer. A first guess can be obtained as follows: if two sources of almost equal rates are used and the background is omitted, Eq. 5 reduces to

$$r_{12}e^{2n\tau} = 2re^{n\tau};\ n_1 \approx n_2 = n;\ n_b = 0 \text{ and}$$

$$\tau_0 = \frac{r_{12}}{2r^2} - \ln\frac{2r}{r_{12}}. \qquad (6)$$

The following results have been obtained with a gamma camera (Picker) linked to a computer (DEC-PDP 11/20). Four point sources (2 mCi ^{99m}Tc) in about a 2-ft distance were measured for 20 sec in the following combinations:

$$r_A = 243{,}323 \text{ counts}$$
$$r_B = 208{,}257 \text{ counts}$$
$$(r_A + r_B) = r_1 = 363{,}189 \text{ counts}$$
$$(r_\tau + r_0) = r_2 = 380{,}913 \text{ counts}$$
$$(r_1 + r_2) = 503{,}895 \text{ counts}$$
$$r_b = 1{,}082 \text{ counts/100 sec}$$
$$r_A,r_B:\ \tau_0 = 15.518\ \mu\text{sec} \quad \tau = 15.584\ \mu\text{sec}$$
$$r_1,r_2:\ \tau_0 = 14.190\ \mu\text{sec} \quad \tau = 14.193\ \mu\text{sec}$$

Method 2 from the article by Adams and Zimmerman would give the following results:

$$r_A,r_B:\ \tau^* = 21.5\ \mu\text{sec}$$
$$r_1,r_2:\ \tau^* = 25.6\ \mu\text{sec}$$

It should be pointed out again that the complexity of data acquisition equipment may influence strongly these calculations. If some intermediate storage of only a few events ($p = 3;5$) should allow queuing and, therefore, averaging of time intervals, Eq. 1 must be evaluated accordingly. Deadtime of a device should not depend on its imput signal.

MANFRED HUTTIG
C. H. F. Muller GmbH
Hamburg 2, Germany

REFERENCES

1. ADAMS R, ZIMMERMAN D: Methods for calculating the deadtime of Anger camera systems. *J Nucl Med* 14: 496–498, 1973

2. FÜNFER, NEUERT: *Zählrohre und Szintillations-zähler.* Karlsruhe, 1959

THE AUTHORS' REPLY

In the definition and measurement of deadtime, two extreme limiting types of deadtime performance may be considered (*1,2*):

1. Paralyzable. This type of equipment remains insensitive for an elapsed "deadtime" τ after each "true" event. The response time τ to an initial event is further extended for an additional time τ by any additional true events which occur before full recovery takes place. This type of equipment is characterized by Poisson statistics. For such paralyzable systems the true counting rate is

$$n = re^{n\tau}. \quad (1)$$

where r is the measured counting rate, and τ is the deadtime as defined for a paralyzable system. The maximum measured counting rate occurs when $n\tau = 1$, or when $r = 1/\tau e$. For large values of true counting rate the observed counting rate decreases and approaches zero.

2. Nonparalyzable. This type of equipment remains insensitive for an elapsed deadtime T after each *observed* event. The deadtime is not affected by any additional true events which occur before full recovery takes place. For such nonparalyzable systems the true counting rate is

$$n = \frac{r}{1 - rT}. \quad (2)$$

where T is the deadtime as defined for a nonparalyzable system. The above expression is exact and not an approximation. At very large true counting rates the measured counting rate plateaus and approaches 1/T.

Most scintillation cameras and associated data processing equipment provide overall performance somewhat intermediate between the limits of paralyzable and nonparalyzable systems (*2*). Although at very low counting rates Eq. 1 and Eq. 2 yield nearly identical values, deadtime must be precisely defined and measured for the correction of rapid dynamic quantitative studies performed at high counting rates.

Because of the mathematical simplicity of Eq. 2, we have chosen to treat deadtime performance as a nonparalyzable system (*3*). In order to do so, however, the deadtime must be considered a dependent variable of the measured counting rate r. This type of treatment results in quite precise correction of histogram curves for deadtime losses.

We congratulate Mr. Huttig on his excellent method to calculate, from two-source data, the deadtime of a paralyzable system. We have written a BASIC program which includes his first approximation equation (6) and a five-dimensional Newton-Raphson iteration method to converge on the four true counting rates and the deadtime of two-source method data, as contained in Eq. 5. This program precisely verifies Mr. Huttig's numerical results. We have also applied this program to data from a number of scintillation camera systems and find that deadtime values so obtained are more nearly independent of counting rate than with the quadratic Eq. 3. However, once the value of deadtime is calculated by this method, correction of counting rate data must be performed by Eq. 1, solution of which is complicated by the presence of the unknown quantity n appearing in the exponential.

Assume a system measuring a true counting rate n = 30,000/sec and a measured rate r = 20,000/sec. For a nonparalyzable system, the deadtime $T = \frac{n - r}{rn} = 16.667\ \mu sec$. For a paralyzable system the deadtime $\tau = \frac{\ln (n/r)}{n} = 13.515\ \mu sec$.

If n is unknown and is to be calculated from r:

For the nonparalyzable system:

$$n = \frac{r}{1 - rT} = \frac{20{,}000}{1 - 20{,}000 \times 16.667 \times 10^{-6}} = 30{,}000.$$

For the paralyzable system, n can be approached by iteration:

$$n_1 = re^{r\tau} = 20{,}000\ e^{(20{,}000 \times 13.515 \times 10^{-6})} = 26{,}207$$

$$n_2 = Re^{n_1\tau} = 28{,}500$$

etc.

$$n_9 = Re^{n_8\tau} = 29{,}996.$$

Other mathematical schemes are available to converge on the answer more rapidly.

The foregoing discussion, in which we have discussed two equally valid but quite different definitions of deadtime, illustrates the present ambiguity of the term "deadtime". "Deadtime" is quite meaningless unless the definition, measurement parameters, and method of calculation are prescribed.

We urge that a committee of the Society of Nuclear Medicine define "deadtime" and standardize methods for its measurement.

On request, the authors will be pleased to provide a listing of the BASIC Newton-Raphson iteration program.

RALPH ADAMS
DUANE ZIMMERMAN
Loma Linda University
Loma Linda, California

REFERENCES

1. EVANS RD: Applications of Poisson Statistics to Some Instruments Used in Nuclear Physics. *The Atomic Nucleus*, New York, McGraw-Hill, 1955, pp 785–788

2. BUDINGER TF: High Counting Rate Performance of the Anger Scintillation Camera: Lawrence Berkeley Lab Report No 2145, 1973

3. ADAMS R, ZIMMERMAN D: Methods for calculating the deadtime of Anger camera systems. *J Nucl Med* 14: 496–498, 1973