QUANTITATIVE EFFECTS OF STATIONARY LINEAR IMAGE PROCESSING ON NOISE AND RESOLUTION OF STRUCTURE IN RADIONUCLIDE IMAGES

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Processing radionuclide image data in an attempt to improve its diagnostic usefulness inevitably changes both the noise and the resolution of structures in the image. Although at present little conclusive evidence exists to indicate whether or not image processing is beneficial, an understanding of the quantitative effects of various processing techniques on parameters of resolution, noise magnitude, and noise texture seems to provide a useful beginning towards solution of the more difficult problem of understanding the effects of image processing on diagnostic image quality. Quantitative effects of stationary linear image processing procedures, which include all convolutions and "filtering" operations, can be predicted rather simply. In the present work, expressions for predicting these quantitative effects are developed and are applied to computer-synthesized image data to provide illustrative examples of the effects of typical noise-smoothing and resolution-enhancing operations.

Image processing changes both the noise and the resolution of structures in images. Smoothing the image of a radionuclide distribution to reduce noise, for example, also tends to degrade resolution and to change the "texture" of the remaining noise. Similarly, processing to improve spatial resolution tends to increase image noise and to change its texture. In order to understand the effects of various image processing techniques on image quality, it seems reasonable to expect that we must understand their quantitative effects on both resolution and noise. Although the relationship between image quality in the clinical sense and quantitative measures of resolution and noise is not at all clear at present, an understanding of the quantitative effects of various processing techniques seems to be a useful beginning towards solution of the more difficult problem of understanding the effects of image processing on diagnostic image quality.

It is the purpose of this paper to point out fundamental mathematical relationships which enable one to predict the effects of stationary linear processing procedures on measures of spatial resolution, noise magnitude, and noise texture" or "character." Stationary linear image processing techniques (*1–11*) include all operations on the data describing the image that can be achieved by replacing each image element value by a weighted linear combination of the element of interest and surrounding elements (linearity), in which the combination rule employed (involving addition, subtraction, and/or differentiation) is independent of position on the image plane (stationarity or shift-invariance). Such techniques are particularly amenable to theoretical analysis because Fourier techniques can be used to evaluate resolution and because their effects on image noise statistics can be computed rather simply. For this reason, stationary linear processing techniques have been studied extensively compared with nonstationary and nonlinear radionuclide image-processing techniques, despite the fact that the latter methods appear promising because of their versatility and adaptability to local conditions of noise level and object structure (*12*). Unfortunately, analysis and generalization of the effects of nonlinear and nonstationary operations on spatial resolution and noise are difficult and will not be attempted here.

The following discussion will be directed primarily to the case of digital image processing, in which the unprocessed image data are available in the form of a two-dimensional matrix of discrete image elements, since digital computers appear to afford the most

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versatile means for quantitative image processing research. All of the results to be presented here can be applied also to the case of stationary linear analog image processing (e.g., optical spatial filtering), however, in the limit as the digital image element size is reduced essentially to zero (*13–15*).

FORMULATION

The data corresponding to any single digital radionuclide image can be described by a two-dimensional matrix of random variables $b{:}_{ij}$, each representing the observed density of counts in the square image element (i,j) having side ϵ and area ϵ^2. Hence $b{:}_{ij} \equiv n{:}_{ij}/\epsilon^2$, where $n{:}_{ij}$ is a random variable representing the observed number of counts in the (i,j) element of the spatially quantized image. The image will be described here in terms of count density rather than counts per element to facilitate the analysis for variable image element size and for the limiting case of $\epsilon \rightarrow 0$.

If the detected counts are statistically independent, then the random variables $n{:}_{ij}$ will obey the Poisson probability distribution (*16*), with mean and variance n_{ij}. Consequently, the image matrix count density variables $b{:}_{ij}$ will have mean values $b_{ij} = n_{ij}/\epsilon^2$ and variance $b_{ij}/\epsilon^2 = n_{ij}/\epsilon^4$.

If we assume that the image elements are small enough relative to the width of the imaging system point-spread function to allow spatial frequency aliasing errors to be neglected (*4*), then the expected or mean count density b_{ij} in element (i,j) of the $M \times M$ image matrix will be given by (*4,17*):

$$b_{ij} = S_0\tau \sum_{k,l=1}^{M} a_{k,l}s'_{i-k,j-l}\epsilon^2 \qquad (1)$$

in which S_0 represents plane sensitivity of the imaging system, τ represents imaging time (for a fixed detector) or imaging time per unit area (for a moving detector), a_{kl} represents the effective two-dimensional radioactivity distribution or object to be imaged, s'_{mn} represents the imaging system point-spread function (*17*), which we assume to be independent of position on the image plane, and the summation expression represents convolution of the point-spread function with the activity distribution. Essentially, then, the expected count density distribution is proportional to the activity distribution blurred by the system point-spread function. The discrete Fourier transform (*4*) of the expected image count density matrix is given by

$$B(\nu_x,\nu_y) \equiv \sum_{k,l=1}^{M} b_{k,l}e^{-j2\pi\epsilon(k\nu_x+l\nu_y)}\epsilon^2$$
$$= S_0\tau A(\nu_x,\nu_y)S'(\nu_x,\nu_y) \qquad (2)$$

in which $S'(\nu_x,\nu_y)$ and $A(\nu_x,\nu_y)$ represent the imaging system transfer function (*18*) and the discrete Fourier transform of the radioactivity distribution, respectively. Note that two-dimensional fast Fourier transformation (*19,20*) of the matrices $\{b_{ij}\}$ and $\{a_{ij}\}$ yields $(1/\epsilon^2)$ times the functions $B(\nu_x,\nu_y)$ and $A(\nu_x,\nu_y)$ evaluated at the discrete frequency values $\nu_x = \pm m/M\Delta x$ and $\nu_y = \pm n/M\Delta x$ (m and n = 0, 1, . . . , M/2).

One can show that any stationary linear image processing operation is equivalent to a convolution of the observed noisy image $\{b{:}_{ij}\}$ with an appropriate processing point-spread function $\{t_{mn}\}$ or weight matrix $\{w_{mn}\}$. Hence one can write

$$c{:}_{ij} = \sum_{k,l=1}^{M} b{:}_{kl}t_{i-k,j-l}\epsilon^2$$
$$= \sum_{k,l=1}^{M} b{:}_{kl}w_{i-k,j-l} \qquad (3)$$

for the noisy processed image matrix $\{c{:}_{ij}\}$ after "correction" by any stationary linear processing procedure. In this expression, the matrix $\{t_{mn}\}$ represents relative count density dispersal per unit area and for digital processing equals a relative weight matrix $\{w_{mn}\}$ divided by ϵ^2. If the radionuclide image data are subjected to a sequence of stationary linear procedures, then the processing point-spread function matrix $\{t_{mn}\}$ or weight matrix $\{w_{mn}\}$ in the above expression should be interpreted as the joint convolution of the individual processing spread functions or weight matrices. The result will be independent of the order in which the procedures are applied since convolution operations are commutative.

RESOLUTION AFTER PROCESSING

It is convenient for the purposes of the present discussion to evaluate the resolution of structures in the processed image in terms of resolution in the expected or mean processed image matrix (which can be thought of either as the "noiseless" image proportional to a processed image produced with an arbitrarily large number of counts or the average of repeated similar images), and to discuss separately* the statistical fluctuations, or "noise," in any single image.

Using Eqs. 1 and 3, one can show (*4*) that the expected processed image $\{c_{ij}\}$ is given by convolution of the radioactivity distribution $\{a_{ij}\}$ with

* This is in contrast to the approach taken by Vetter and Pizer (*21*) in which "resolution" is defined as a Euclidean sum of "geometric" and "statistical" terms. We discuss "geometric" resolution here.

an overall processed point-spread function $\{u_{ij}\}$ which is the joint convolution of the processing and imaging system spread functions:

$$c_{ij} = \sum_{k,l=1}^{M} b_{kl} t_{i-k,j-l} \epsilon^2 = \sum_{k,l=1}^{M} b_{kl} w_{i-k,j-l} = S_0 \tau \sum_{k,l=1}^{M} a_{kl} u_{i-k,j-l} \epsilon^2 \quad (4)$$

in which

$$u_{ij} = \sum_{k,l=1}^{M} s'_{kl} t_{i-k,j-l} \epsilon^2 = \sum_{k,l=1}^{M} s'_{kl} w_{i-k,j-l}. \quad (5)$$

In the spatial frequency domain, the discrete Fourier transform $C(\nu_x,\nu_y)$ of the expected processed image is given by

$$\begin{aligned} C(\nu_x,\nu_y) &= B(\nu_x,\nu_y)T(\nu_x,\nu_y) \\ &= S_0\tau A(\nu_x,\nu_y)\,[S'(\nu_x,\nu_y)T(\nu_x,\nu_y)] \\ &= S_0\tau A(\nu_x,\nu_y)U(\nu_x,\nu_y). \end{aligned} \quad (6)$$

Hence resolution in the expected processed image can be evaluated in terms of either the overall processed point-spread function $\{u_{ij}\}$ or the overall processed transfer function $U(\nu_x,\nu_y)$.

A general goal of various resolution enhancement schemes is to make the overall processed spread function $\{u_{ij}\}$ "narrower" in some sense than the imaging system spread function $\{s'_{ij}\}$ by proper choice of the processing spread function $\{t_{mn}\}$ or weight matrix $\{w_{mn}\}$. In the spatial frequency domain, the equivalent goal is to keep the overall transfer function $U(\nu_x,\nu_y)$ approximately constant to higher spatial frequencies than is $S'(\nu_x,\nu_y)$. This can be accomplished by choosing $T(\nu_x,\nu_y) \approx 1/S'(\nu_x,\nu_y)$, at least over a range of frequencies, but the price paid is a change in image noise texture and sometimes an increase in the magnitude of image noise as we shall show.

Alternatively, a general goal of noise-smoothing schemes is reduction of image noise magnitude by suppression of high frequency components through appropriate choice of the processing point-spread function $\{t_{mn}\}$ or weight matrix $\{w_{mn}\}$. Such techniques tend to reduce response of the overall transfer function at high frequencies, however, and tend to increase the width of the overall spread function correspondingly.

NOISE MAGNITUDE AFTER PROCESSING

The magnitude or "intensity" of statistical fluctuations in the processed image can be measured by the "variance" of observed count densities in any image element in a large number of repeated images of the same radioactivity distribution. The "standard deviation" of statistical fluctuations in count density is defined as the square root of count density variance and is equivalent to the root-mean-square fluctuation in count density in an element of repeated images.

Because of the Poisson nature of unprocessed image statistics, count density variance in the processed image may vary from point to point depending upon local expected count density. In general, processed count density variance is given by convolution of the expected unprocessed image with the square of the processing point-spread function or weight matrix (*4*):

$$\mathrm{Var}\,\{c{:}_{ij}\} = \sum_{k,l=1}^{M} b_{kl}[t_{i-k,j-l}]^2\epsilon^2 = \sum_{k,l=1}^{M} b_{kl}[w_{i-k,j-l}]^2. \quad (7)$$

In regions where local expected count density is approximately uniform,

$$\mathrm{Var}\,\{c{:}_{ij}\} \doteq b_{ij} \sum_{k,l=1}^{M} [t_{k,l}]^2\epsilon^2 = \mathrm{Var}\,\{b_{ij}\} \sum_{k,l=1}^{M} [w_{k,l}]^2. \quad (8)$$

This result can also be expressed in terms of spatial frequency response of the processing procedure by

$$\mathrm{Var}\,\{c{:}_{ij}\} \doteq b_{ij} \iint_{-1/(2\epsilon)}^{+1/(2\epsilon)} |T(\nu_x,\nu_y)|^2 d\nu_x d\nu_y = \mathrm{Var}\,\{b{:}_{ij}\} \cdot \frac{1}{M^2} \sum_{k,l=-(M/2-1)}^{M/2} |W_{k,l}|^2 \quad (9)$$

where $\{W_{k,l}\}$ are the filter values used to multiply the FFT of the original data before inverse fast Fourier transforming to produce the processed image. Note that the second forms of Eqs. 8 and 9 provide simple means of predicting output variance relative to input variance after digital processing.

We remark that linearly processed image statistics tend toward the Gaussian probability distribution due to the central limit theorem (*22*) and that in this case the resulting standard deviation can be interpreted in the conventional way as denoting approximately 0.68 probability of fluctuations occurring within ± 1 s.d., etc. Note, however, that processed image statistics are *not* Poisson distributed.

One can see from Eqs. 8 and 9 that the use of a processing spread function $\{t_{mn}\}$ or digital weight matrix $\{w_{mn}\}$ corresponding to a processing transfer function $T(\nu_x,\nu_y)$ or filter $\{W_{kl}\}$ which assumes values greater than unity may cause count density variance in the processed image to be greater than unprocessed variance, b_{ij}/ϵ^2. Hence, processing to enhance resolution using a transfer function or filter $\approx 1/S'(\nu_x,\nu_y)$ can increase noise magnitude. Similarly, use of a "low pass" processing transfer function or filter which falls rather quickly to zero will reduce noise magnitude*. Resolution enhancement techniques which are practical for noisy image data must combine enhancement at some (usually low) frequencies with suppression at other (usually high) frequencies; the effect on noise magnitude will depend on the chosen transfer function or filter through Eqs. 8 and 9.

NOISE CHARACTER AFTER PROCESSING

The character or "texture" of image noise can be expressed in terms of count density "autocovariance" which is defined as the expected product of deviations in count density from the respective mean values at any two points (*4,12,22*):

$$\text{Covar}\,\{c\!:_{ij}, c\!:_{i+m,j+n}\} \equiv E\{[c\!:_{ij} - c_{ij}][c\!:_{i+m,j+n} - c_{i+m,j+n}]\}. \quad (10)$$

At zero separation of the two points [i.e., $(m,n) = (0,0)$], autocovariance reduces to variance.

Generally speaking, a noise autocovariance function which drops rapidly to zero as the distance between points increases indicates "sharp" noise, with deviations from respective means, or noise outcomes, essentially independent except at small distances; on the other hand, an autocovariance function which falls slowly to zero as the distance between points increases indicates the presence of broad noise undulations, with noise outcomes at neighboring points highly correlated. Positive values of the autocovariance function at some separation distance indicate that noise outcomes tend to be of the same sign (+ or −) at that separation. Negative autocovariance values indicate expected noise outcomes of opposite sign at that separation distance, tending to cause the noise to appear "ragged" with somewhat regular changes from positive to negative fluctuations at that separation distance. The count density autocovariance function plotted against separation distance thus in a sense describes the "shape" of the noise and hence the "shape" of possible processed image noise artifacts. Note that since radionuclide image noise is nonstationary except for uniform objects, conventional "Wiener" or "power" spectra (*22,23*) cannot be used to describe noise character, except qualitatively, to the extent that normalized autocovariance functions are approximately shift invariant.

Radionuclide image count density autocovariance resulting from the application of any stationary linear processing procedure is given by the convolution of the expected input $\{b_{ij}\}$ with the product of the processing spread function matrix $\{t_{kl}\}$ and the processing matrix shifted appropriately $\{t_{k+m,l+n}\}$; that is,

$$\text{Covar}\,\{c\!:_{ij}, c\!:_{i+m,j+n}\} = \sum_{k,l=1}^{M} b_{kl}[t_{i-k,j-l}t_{i-k+m,j-l+n}]\epsilon^2. \quad (11)$$

In locally uniform regions of the expected input image, output autocovariance is given approximately by:

$$\text{Covar}\,\{c\!:_{ij}, c_{i+m,j+n}\} \doteq b_{ij}\sum_{k,l=1}^{M} t_{k,l}t_{k+m,l+n}\epsilon^2 = \text{Var}\,\{b\!:_{ij}\}\sum_{k,l=1}^{M} w_{k,l}w_{k+m,l+n} \quad (12)$$

in which the second factor on the right represents convolution of the processing spread function or weight matrix with a version of itself, reflected about its origin. Hence a measure of the texture of processed image noise is provided by plotting Eq. 12 as a function of displacement (m,n), normalized to unity at $(0,0)$.

It should be clear from Eq. 12 that if the processing spread function or weight matrix contains only positive or zero elements, then the corresponding autocovariance function will be somewhat broader and will contain only positive or zero elements also. Hence, the noise texture will be rather smoothly undulating with undulations comparable in width to the autocovariance function width. If the processing spread function contains some negative elements, however, as any resolution enhancing spread function must, then the corresponding autocovariance function can assume negative values also, causing the processed noise to appear "ragged" with somewhat regular positive to negative trends at the displacement of the negative autocovariance values.

EXAMPLES

In order to illustrate the effects of stationary linear image processing discussed above, an image of two

* One can show that any processing spread function $[t_{mn}]$ or weight matrix $[w_{mn}]$ (other than the identity spread function which leaves the data unchanged) which contains only positive or zero elements must reduce relative noise magnitude. Processing spread functions or weight matrices which contain negative elements may increase or decrease noise magnitude depending on the value of the second factor of Eqs. 8 and 9. One can also show that any transfer function $T(\nu_x,\nu_y)$ or filter $[W_{kl}]$ assuming values greater than the value at zero frequency (unity) must correspond to a spread function $[t_{mn}]$ or weight matrix $[w_{mn}]$ containing some negative elements.

point sources 1.4 cm apart was simulated using a digital computer. The expected unprocessed image was synthesized assuming a Gaussian imaging system point-spread function with full width at half maximum (FWHM) of 1 cm, a digital image element size of 0.2 cm $\times$ 0.2 cm, a 128 $\times$ 128 element image matrix, an expected background count density of 625 counts/cm^2, and an expected count density of 875 counts/cm^2 directly over each point source. Poisson-distributed random variables were generated about the appropriate expected element values to yield a simulated noisy image. Expected and observed noisy image element values in a row passing through the two point source locations are shown in Fig. 1A together with the ± 1 s.d. bounds and, as an inset, the unprocessed image autocovariance function normalized to unity at zero separation distance.

The simulated noisy image was then processed by two different stationary linear techniques using a fast Fourier transform (FFT) algorithm. The two processing methods and the associated processing parameters were selected to produce results which would clearly exemplify the results discussed above and are not necessarily optimal in any sense.

Figure 1B shows the noisy data of Fig. 1A after processing with a Gaussian point-spread function having FWHM = 0.6 cm (3 elements). Also shown in Fig. 1B is the expected processed image computed using Eq. 4, the ± 1 s.d. bounds computed using Eq. 7, and, as an inset, the processed image autocovariance given by Eq. 12 after normalization to unity at zero separation distance. Clearly, noise magnitude is reduced markedly, resolution of the point sources is reduced somewhat, and noise texture is made rather smoothly undulating, with undulations comparable in width to that of the normalized autocovariance function.

Figure 1C shows the noisy data of Fig. 1A after processing with the filter (*4*)

$$T(\nu_x,\nu_y) = Q^{(n)}(\nu_x,\nu_y) = \frac{1 - (1 - |S'(\nu_x,\nu_y)|^2)^{n+1}}{S'(\nu_x,\nu_y)} \quad (13)$$

using n = 4 and the assumed imaging system transfer function for $S'(\nu_x,\nu_y)$. Spatial frequency response of this filter rises from unity at low spatial frequencies, approximating the inverse of the imaging system transfer function, reaches a maximum response of about 1.5 near 0.5 cycles/cm; and then falls smoothly toward zero, assuming a value of 0.1 at 1.1 cycles/cm. Hence this filter combines low frequency correction for resolution degradation by the original imaging system with high frequency noise suppression. Comparison of the original and the processed data in this case shows that noise magni-

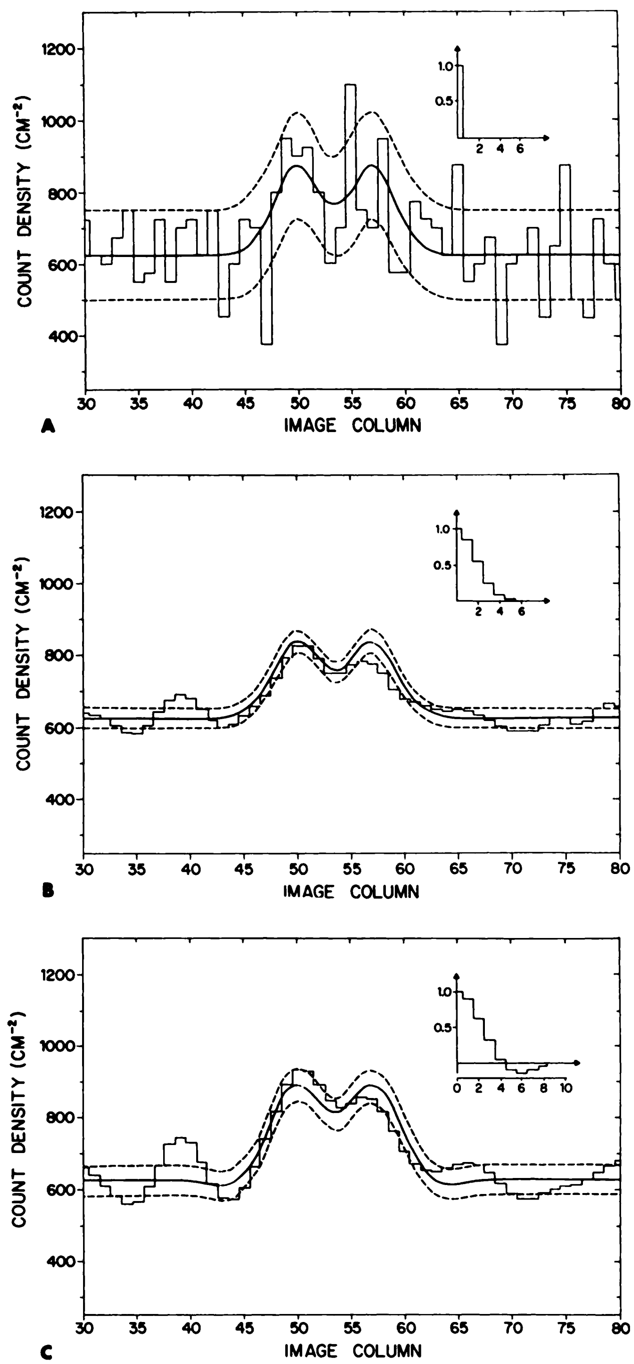

FIG. 1. Count density profiles through image of two point sources. (A) Unprocessed image simulated assuming Gaussian imaging system point-spread function with FWHM = 1.0 cm (5 image elements). (B) Simulated data after processing with Gaussian smoothing function with FWHM = 0.6 cm (3 elements). (C) Simulated data after processing with filter described by Eq. 13 with n = 4. In each case, expected image is shown as smooth solid curve ± 1 s.d. bounds are shown as smooth broken curves; and noisy observed image is shown as stepped curve. Image autocovariance function given by Eq. 12, normalized to unity at its origin and plotted against separation distance, is shown inset on each graph.

tude is reduced markedly, resolution, and in particular, contrast in the image of the point sources is increased slightly, and noise texture is made somewhat oscillatory or "ragged," consistent with the computed autocovariance function shown as an inset. Use of a larger value of n in the filter of Eq. 13 tends to cause noise magnitude to be reduced less and finally to be increased, to increase resolution of the point sources, and to increase the oscillatory nature of the processed noise texture. Note that this enhancement procedure tends to emphasize noise artefacts such as that centered near element 39 in addition to real structure, as one might expect.

DISCUSSION

The degree to which image quality depends on resolution, noise magnitude, and noise character seems to depend in a complex way both upon the source distribution and upon the information sought from the image. If the activity distribution is inherently unsharp, for example, then high spatial resolution in data acquisition and processing may be of little importance, and usefulness of the image will depend primarily upon noise character and noise magnitude in the final result. If the source structure of interest is small, however, or if definition of edges or boundaries is critical, then good resolution may be required, and resolution enhancement may prove beneficial, even at the expense of *some* noise enhancement. If a processing procedure results in noise of a certain texture, then usefulness of the method will depend on whether or not that noise texture can be confused with potentially relevant object structure; "mottled" noise might be interpreted as diffuse metastases or cirrhosis in a radionuclide image of the liver, for example, but possibly not as real structure in an image of the brain.

We have attempted here to point out methods for computing the quantitative effects of stationary linear image-processing techniques on such fundamental parameters as resolution, noise magnitude, and noise character. These methods may be used to compare different techniques and thus to guide the design of processing schemes for specific clinical situations. The usefulness of rationally designed processing procedures must ultimately be decided empirically by carefully controlled and analyzed perception experiments involving human observers.

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REFERENCES

1. Sprau AC, Tauxe WN, Chaapel DW: A computerized radioisotope scan-data filter based on a system response to a point source. *Mayo Clin Proc* 41: 585–598, 1966

2. Iinuma TA, Nagai T: Image restoration in radioisotope imaging systems. *Phys Med Biol* 12: 501–509, 1967

3. Hart RW, Farrell RA: Imaging distribution of radioactivity within the human body. I. Theoretical considerations in optimum data processing. *Invest Radiol* 3: 199–212, 1968

4. Metz CE: *A Mathematical Investigation of Radioisotope Scan Image Processing*. Doctoral thesis, University of Pennsylvania, 1969. Ann Arbor, Mich, University Microfilms, Publication 70-16-186.

5. Nagai T, Fukuda N, Iinuma TA: Computer-focusing using an appropriate Gaussian function. *J Nucl Med* 10: 209–212, 1969

6. Hunt WA, Lorenz WI, Meder HG, et al: *Digital Processing of Scintigraphic Images*. Wissenschaftliches Zentrum, Heidelberg, Germany, IBM Tech. Rept. 70.03.001, 1970

7. Metz CE: On the differential method of image focusing. *J Nucl Med* 11: 142–143, 1970

8. Tanaka E, Iinuma TA: Approaches to optimal data processing in radioisotope imaging. *Phys Med Biol* 15: 683–694, 1970

9. Barber DC, Mallard JR: Data processing of radioisotope images for optimum smoothing. *Phys Med Biol* 16: 635–644, 1971

10. Brown DW, Kirch DL, Ryerson RW, et al: Computer processing of scans using Fourier and other transformations. *J Nucl Med* 12: 287–291, 1971

11. Kirch DL, Brown DW: Recent advances in digital processing static and dynamic scintigraphic data. In *Proc. 2nd Oak Ridge Symposium on Sharing of Computer Programs and Technology in Nuclear Medicine*, April 1972, Springfield, Va, National Technical Information Services, US Dept of Commerce, USAEC Conference Series

12. Metz CE, Pizer SM: Nonstationary and nonlinear scintigram processing. In *Proc. 2nd International Conference on Data Handling and Image Processing in Scintigraphy*, Hannover, Germany, October 1971, Todd-Pokropek AE, Jahns E, eds, 1974, in press

13. Tippett JT, Berkowitz DA, Clapp LC, et al, eds: *Optical and Electro-optical Information Processing*, Cambridge, Mass, MIT Press, 1965

14. Beck RN, Harper PV, Charleston DB, et al: Optical processing of radionuclide images. In *Biomedical Sciences Instrumentation*, vol 6, Pittsburgh, Pa, Instrument Society of America, 1969, pp 241–247

15. Cykowski CB, Kirch DL, Pothemus CE, et al: Image enhancement of radionuclide scans by optical spatial filtering. *J Nucl Med* 12: 85–87, 1971

16. Haight FA: *Handbook of the Poisson Distribution*, New York, John Wiley, 1967

17. Beck RN, Zimmer LT, Charleston DB, et al: Advances in fundamental aspects of imaging systems and techniques. In *Medical Radioisotope Scintigraphy*, Vienna, IAEA, 1973

18. Beck RN: Nomenclature for Fourier transforms of spread functions of imaging systems used in nuclear medicine. *J Nucl Med* 13: 704–705, 1972

19. Cooley JW, Tukey JW: An algorithm for the machine calculation of complex Fourier series. *Math Compt* 19: 297–301, 1965

20. Cochran WT, Cooley JW, Favin DL, et al: What is the fast Fourier transform? *Proc IEEE* 55: 1664–1674, 1967

21. Vetter HG, Pizer SM: A measure for radioisotope scan image quality. *J Nucl Med* 12: 526–529, 1971

22. Papoulis A: *Probability, Random Variables, and Stochastic Processes.* New York, McGraw-Hill, 1965

23. Blackman RB, Tukey JW: *Measurement of Power Spectra*, New York, Dover, 1959