RADIONUCLIDE TOMOGRAPHIC IMAGE RECONSTRUCTION USING FOURIER TRANSFORM TECHNIQUES

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Transverse tomograms have been generated from gamma camera pictures by the implementation of a Fourier reconstruction technique. Given a set of projections of a radioactive object obtained from various angles around a central axis, it can be shown that the Fourier transforms of these projections yield a sampled representation of the Fourier transform of the activity distribution of the object. From this it is possible to reconstruct a tomogram of the object by processing and inverse transforming the sampled frequency function. Tomograms reconstructed by this technique are of high quality, demonstrating spatial accuracy and quantitative accuracy within limits imposed by internal absorption, detector resolution, and counting statistics.

There is great interest in developing a method of nondestructively determining the internal structure of three-dimensional objects from external measurements. A typical problem is to determine the distribution of the absorption coefficients in an object measured by a transmitted beam of radiation such as x-rays or electrons (*1–3*). Objects which are self-luminous; e.g., radioactive, can also be imaged in terms of their internal radiant intensity distribution. Problems such as these are found in many fields such as electron microscopy, radiology, radioastronomy, and nuclear medicine. Although such diverse applications suggest separate problems, the actual difference lies only in the measured quantities and the approach followed in gathering the measurements. The problem may thus be considered as one of "three-dimensional imaging techniques" independent of the type of image being reconstructed. Our use of the term "three-dimensional imaging" will imply the concept of both spatial and quantitative accuracy in the final reconstruction.

The concept of three-dimensional imaging need not be limited to techniques which reconstruct three-dimensional images as a gestalt. A set of contiguous, serial tomographic images of an object in effect constitutes a three-dimensional image if each tomogram contains data only from the corresponding section of the object. This approach simplifies the situation both conceptually and operationally by reducing the reconstruction problem from three to two dimensions and also solves the problem of displaying a three-dimensional image having internal structure without obscuring that structure from clear observation.

The form of tomography known as transverse axial or transverse section tomography (TST) as exemplified by the EMI Scanner for transmission imaging (*1*) and the work of Kuhl, et al for emission imaging (*4*) yields section images which are independent of each other in terms of data content. Thus a technique yielding quantitatively accurate transverse section tomograms also represents a solution to the problem of three-dimensional imaging.

Among the many known techniques for reconstructing TSTs (*5,6*) is one which we shall call the Fourier transform technique (FTT). In addition to having a sound theoretical foundation, this and related "noniterative" techniques have certain apparent advantages which make them particularly attractive for clinical use (*7*).

The FTT was independently discovered by several groups of investigators in diverse fields. Bracewell (*8*) originally proposed the technique in radioastronomy in 1956. In 1968 DeRosier, et al (*2*) published their work on the application of the technique to electron microscopy and in 1969 Tretiak, et al proposed and demonstrated the same method for the reconstruction of x-ray tomograms (*3*). The work of these authors has demonstrated the validity

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and usefulness of the technique for both transmissive and emissive imaging situations.

The considerations discussed above provide a strong justification for the belief that a three-dimensional imaging system based upon the FTT is both possible and practical in nuclear medicine. Preliminary work in our laboratory has also supported this belief (*7,9,10*). In this paper we present a review based on delta function sampling of the theory underlying the FTT and its relationship to some other tomographic techniques and present the results of an implementation of this technique using conventional nuclear medicine instrumentation.

DEFINITIONS

The following terminology will be followed throughout the remainder of the discussion. This represents a slight modification of the terminology proposed by Rowland (*11*).

The data input to the reconstruction process consists of a set of images of the object obtained from various angles around a central axis passing through the object. In the present application this data consists of a set of two-dimensional images of a radioactive object obtained with a standard gamma camera. These images will be called rotation views. The irradiance data in each rotation view lying along a line at right angles to the axis of rotation at the level of the desired tomographic section will be called a projection.

A ray is a region of the picture lying between two parallel lines extending outward normal to the projection. There will be a set of m rays of equal width associated with each projection and covering the entire area of the desired tomogram.

For each projection there will be an angle θ between that projection and the x axis (Fig. 2).

The real ray sum is the measured integral value for each ray of each projection.

THEORY

The FTT is based on the fact that the one-dimensional projections of a source provide information about the spatial frequencies present in the source. Many projections appropriately chosen may then be used to determine the spatial frequency representation of the entire source.

Consider the three-dimensional source depicted in Fig. 1. Its isotopic radiant intensity distribution is given by $a(x,y,z)$. Using a detector with a very narrow field of view, only a section of the source at level z_0 is interrogated during a single scan (Fig. 1).

The orientations of the projections can be related and expressed in terms of a rotated Cartesian coordinate system (x',y') as shown in Fig. 2. The new (x',y') system is rotated by θ degrees with respect to the fixed (x,y) frame of reference.

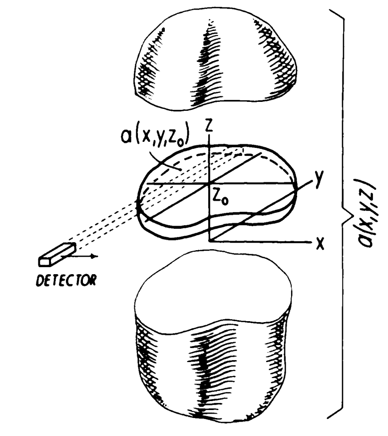

FIG. 1. Hypothetical distribution of radioactivity $a(x,y,z)$ showing coordinate system orientation and relationship of tomographic section $a(x,y,z_0)$ to single-detector element during data acquisition.

A one-dimensional projection along the rotated x′ direction is given by

$$g_\theta(x',z_0) = \int_{-\infty}^{\infty} a_\theta(x',y',z_0)\,dy' \qquad (1)$$

where the detector is idealized to have an extremely narrow field of view (collecting rays only in the y′ direction) and internal absorption losses are ignored (vide infra).

A set of $g_\theta(x',z_0)$ functions result from obtaining many projections of the source function $a(x,y,z_0)$ at incremental values of angular rotation θ. The Fourier transform of each $g_\theta(x',z_0)$ given by $G_\theta(u',z_0)$ provides the Fourier transform of the source function along one radial line in frequency space. This radial line is at the same orientation as the projection. The process is mathematically shown in the formulation below and depicted in Fig. 2, where $A_\theta(u',v',z_0)$ is the two-dimensional Fourier transform of $a_\theta(x',y',z_0)$.

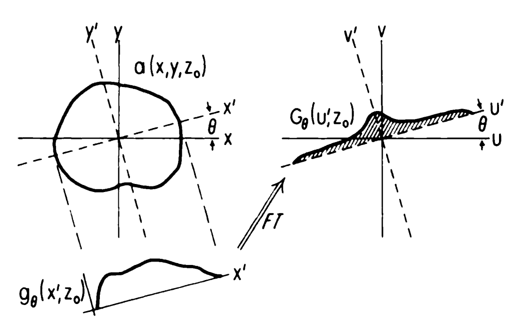

FIG. 2. Tomographic section from Fig. 1 viewed from above showing relationship of any single projection $g_\theta(x',z_0)$ and its corresponding Fourier transform $G_\theta(u',z_0)$ to initial (x,y,z) and rotated (x',y',z) coordinate systems.

$$A_\theta(u',v'=0,z_0) = G_\theta(u',z_0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_\theta(x',y',z_0)\exp^{-i2\pi(x'\cdot u'+y'\cdot 0)}dx'dy' \quad (2)$$

Substituting from Eq. 1 gives

$$G_\theta(u',z_0) = \int_{-\infty}^{\infty} g_\theta(x',z_0)\exp^{-i2\pi(x'\cdot u')}dx' \quad (3)$$

It is seen that setting $v' = 0$ reduces the above expression to a one-dimensional Fourier transformation of the measured projection $g_\theta(x',z_0)$. Physically this means one can find the spatial variations or spatial frequency components in the x' direction separately from those in other directions. Hence the spatial frequency function can be synthesized sequentially, along one line at a time, until its two-dimensional polar form is approximated. At this point the spatial frequency function is known along radial lines in frequency space. Essentially, the spatial frequency function has been sampled in a manner determined by the angle θ between individual rotation views. The value at the origin $A(0,0,z_0)$ will, however, be N times too large. This can be corrected by dividing this point value by N or integrating any single projection to determine the dc component.

The desired tomographic reconstruction is obtained by inverse Fourier transforming the frequency function. Before this can be done a correction must be introduced for the scaling of the spatial frequency components which occur because of the angular sampling in frequency space. This correction can be accomplished in either of two ways.

Consider the polar representation of the two-dimensional inverse Fourier transform:

$$a(x,y,z_0) = \int_0^{2\pi}\int_0^{\infty}\left[\frac{\pi}{N}\cdot\frac{\delta(\phi-n\theta_0)}{\rho}\right]\cdot A(\rho,\phi,z_0)\, e^{i2\pi\rho(x\cdot\cos\phi+y\cdot\sin\phi)}\rho\cdot d\rho\cdot d\phi \quad (4)$$

for $n = 0, 1, 2, \cdots (2N-1)$ and $N = \pi/\theta_0$

It can be seen that the angular sampling function $\delta(\phi - n\theta_0)/\rho$, which carries a π/N weight has introduced an inverse ρ scaling to the frequency components $A(\rho,\phi,z_0)$ which overemphasizes the low-frequency components. In fact the result of performing the inverse transform as written in Eq. 4 is a reconstruction equivalent to the additive transverse section tomograms of Kuhl (*4*). This scaling can be corrected by introducing a ρ filtering in the numerator of Eq. 4 giving

$$a(x,y,z_0) = \int_0^{2\pi}\int_0^{\infty}\left[\frac{\pi}{N}\cdot\frac{\delta(\phi-n\theta_0)}{\rho}\right]\cdot\rho\cdot A(\rho,\phi,z_0)\exp^{i2\pi\rho(x\cdot\cos\phi+y\cdot\sin\phi)}\rho\cdot d\rho\cdot d\phi \quad (5)$$

$n = 0, 1, 2 \cdots (2N-1)$

If the limits of integration are then expanded from $-\infty$ to ∞ the two-dimensional transform becomes

$$a(x,y,z_0) = \int_0^{\pi}\int_{-\infty}^{\infty}\frac{\pi}{N}\cdot\delta(\phi-n\theta_0)\cdot A_\phi(\rho,z_0)\exp^{i2\rho\pi(x\cdot\cos\phi+y\cdot\sin\phi)}|\rho|\cdot d\rho\cdot d\phi \quad (6)$$

$n = 0, 1, 2, \cdots (N-1)$

where

$$A_\phi(\rho,z_0) = \begin{cases} A(\rho,\phi,z_0) & 0 \leqq \rho < \infty \\ A(\rho,\pi+\phi,z_0) & -\infty < \rho < 0 \end{cases}$$

$$a(x,y,z_0) = \sum_n \frac{\pi}{N}\int_{-\infty}^{\infty} A_{n\theta_0}(\rho,z_0)\exp^{i2\pi\rho(x\cdot\cos n\theta_0+y\cdot\sin n\theta_0)}|\rho|\cdot d\rho \quad (7)$$

$n = 0, 1, 2, \cdots (N-1)$

which is a sum of sheet functions (back projections) obtained by one-dimensional inverse transforms in the radial coordinate ρ of $A_{n\theta_0}(\rho,z_0)$ corrected by a ramp filter function $|\rho|$. This is the result of Bracewell, et al (*12*) where the filtering is performed on the sheet functions in real space by convolutions and practical limits of integration, $-M \leqq \rho \leqq M$, replacing the infinite limits. This "convolution method" has certain advantages over the FTT which are discussed below and elsewhere (*7*). A technique described by Chesler (*13*) also appears to represent an implementation of this convolution method.

An alternative to the above correction is to interpolate the angularly sampled information in frequency space to a rectangular grid of points (*14*). This changes the sampling function to a two-dimensional comb function: comb (u/u_0) · comb (v/v_0). No frequency-dependent scaling is introduced and a discrete inverse Fourier transformation in rectangular coordinates may be used. This may be easily achieved by the fast Fourier transform algorithm. The rectangular grid required is given by the Whittaker-Shannon sampling theorem. For a space-limited function of maximum diam 2a, the frequency function need only be known on a rectangular grid in frequency space with sample spacings of ½a. This fact is used by Bracewell to determine theoretically the number of projections required to reconstruct the space function (*12*). If M is the maximum spatial frequency in the source, then as shown in Fig. 3, one can determine all sample values in frequency space with N projections where $N \geqq \pi M 2a$. All reconstructions described below were performed using this latter interpolation technique.

METHODS

A program has been written in FORTRAN-IV to implement this technique on our IBM 360/65 computer. The program produces images with 64 × 64

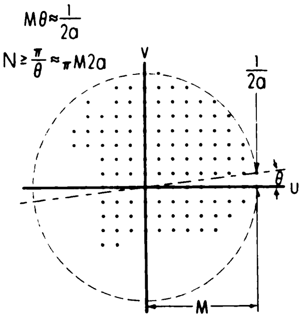

FIG. 3. Schematic representation of relationship between maximum spatial frequency desired in reconstruction, M, and number of equally spaced projections, N, required to achieve this (after Bracewell, et al, 1967).

picture elements and utilizes the fast Fourier transform routine in performing all transformations.

In actual operation the one-dimensional $g_\theta(x',z_0)$ functions are read from each rotation view, Fourier transformed, and the frequency function synthesized in polar form. Next an interpolation is performed to produce a frequency function in rectangular coordinates. At present a simple linear interpolation is being used. The interpolated frequency function is then inverse transformed to yield a transverse tomographic image. Sequential runs can generate many contiguous transverse section tomograms from a single set of gamma camera rotation views.

All data collection was done with a standard commercial gamma camera (Searle Radiographics, Inc. Pho/Gamma HP) and standard collimators. Each camera image (rotation view) was digitized into a 64 by 64 matrix and stored on magnetic disk. A profile (projection) corresponding to the level of the desired tomogram was then selected from each digitized rotation view and these data were transferred to magnetic tape which served as the input for the large computer. The reconstructions were written onto magnetic tape and transferred back to disk for display and analysis on a small digital image acquisition and processing system (Medical Data Systems Corp. "Modumed Trinary System"). This latter system was also used to digitize and store the original rotation views.

A variety of test objects were used to evaluate different aspects of the reconstructions. A capillary tube approximately 1 mm in diameter, filled with radioactivity, was used to evaluate the point-spread function of the system reconstructions including the resolution characteristics of the gamma camera. A group of eight vials of radioactivity in a ring was used to evaluate the effect of multiple sources. A group of three vials of radioactivity, each containing a different amount, was imaged both with and without surrounding absorbing medium to estimate quantitative accuracy and the effects of internal absorption.

Rotation views of each of these objects were obtained for a sufficient length of time to minimize the effects of counting statistics. Typically each projection would contain from 15,000 to 17,000 counts. To simulate a clinical problem a test object was devised consisting of a standard ORINS cylindrical neck phantom containing three vials of activity and positioned within a larger container of radioactive fluid. This represents fairly closely the situation of a patient with multiple metastatic brain tumors. The amount of radioactivity in this phantom was adjusted to simulate clinical counting rates, i.e., on the order of 200,000 cpm. Acquisition time for each rotation view was 30 sec as this seemed a realistic choice for actual patient studies in the future.

RESULTS

Figures 4–7 illustrate reconstructions of each of our test objects by the FTT. These are unprocessed

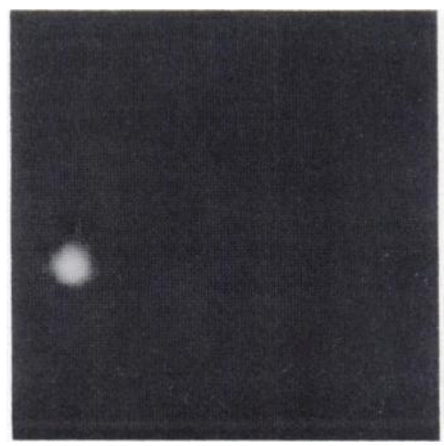

FIG. 4. Transverse section of capillary tube with inside diameter of 1.1 mm. Reconstruction from 60 projections at 6-deg intervals.

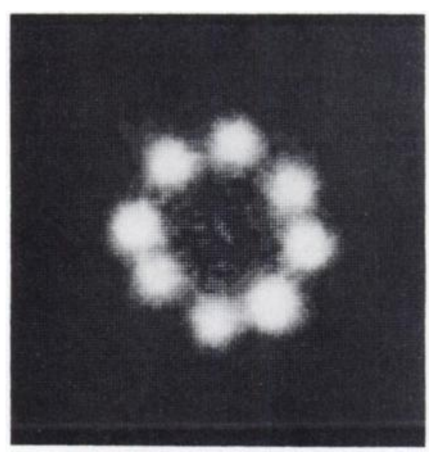

FIG. 5. Transverse section of loose ring of eight vials. Each vial has inside diameter of 14 mm and approximate diameter of circle is 130 mm. Reconstruction from 60 projections at 6-deg intervals.

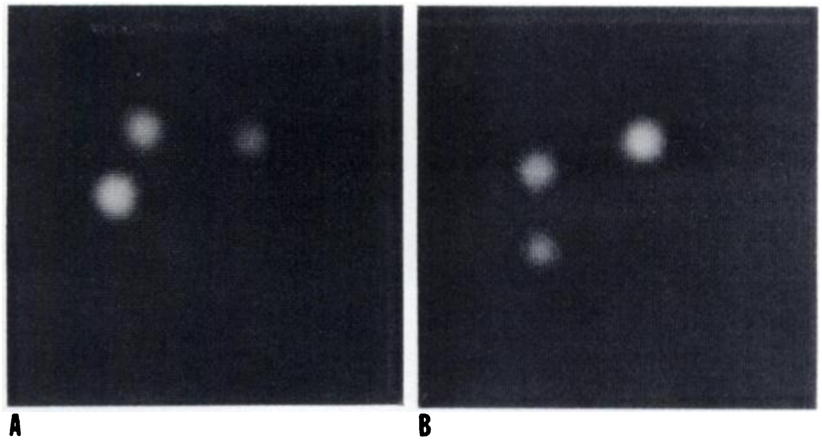

FIG. 6. (A) Reconstruction of three vials (i.d. 14 mm) each containing different concentrations of activity showing preservation of quantitative relationship. Activity ratios between adjacent vials should each be 1.66:1. Measured values in reconstruction are 1.66:1 and 1.71:1. (B) Same as (A) except vials were placed asymmetrically in plastic absorber. Both reconstructions from 60 projections at 6-deg intervals.

reconstructions; i.e., no background subtraction, smoothing, filtering, or contrast enhancement has been used; except that negative numbers have been set to zero to permit the positive portion of the image to be displayed. This was done because our existing display system cannot show both positive and negative numbers simultaneously. Some negative portions of the pictures result from abruptly truncating the infinite spatial frequency function which is necessary to represent accurately a space-bounded function. In practice the frequency function will always be bandlimited. In general the negative numbers are small in magnitude with only a few values reaching an absolute value as high as 10% of the maximum true image value. The mathematical process known as apodization could be used to reduce further the negative portion to smaller values at some expense in resolution.

The spatial resolution of the reconstructions approaches that of the gamma camera itself. Figures 7A and 7B illustrate this. Figure 7A is a top view of our "head" phantom and illustrates what a tomographic reconstruction should look like if the spatial resolution of the gamma camera is preserved. Figure 7B is the actual tomogram.

The reconstructions which we have quantitated show excellent preservation of quantitative relationships except for alterations caused by internal absorption. These alterations in quantitative accuracy are most marked near the center of objects and are almost absent near the periphery and in imaging situations in which little absorbing media is present; e.g., three vials of activity imaged in air.

DISCUSSION

In theory it should require projections only through an arc of 180 deg to reconstruct accurately a tomogram by the FTT. We have chosen to use views through 360 deg for two reasons. This reconstruction algorithm assumes that the rays have parallel sides. In fact they diverge slightly. By averaging each real ray sum by its complement from the opposite side of the object, this divergence is minimized. Secondly this averaging process partially compensates for internal absorption losses.

The images at present are only semiquantitative because the effects of internal absorption and scattering have been ignored. These effects can alter both the quantitative and spatial relationships within the reconstruction and must be corrected if a truly

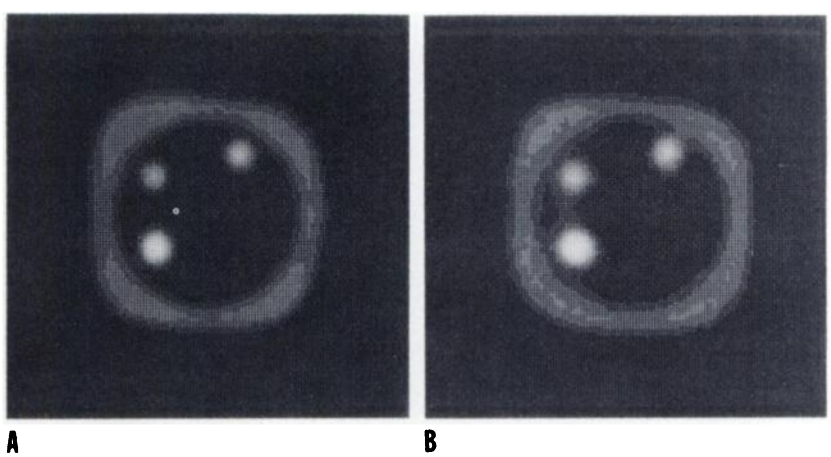

FIG. 7. Head phantom. (A) Camera image taken from above represents "ideal" reconstruction. (B) Tomogram reconstructed from 120 projections at 3-deg intervals.

accurate reconstruction is to be achieved. With our present reconstruction technique the magnitude of these effects appears to be the major limiting factor on quantitative accuracy.

The effects of quantum noise (statistics) on tomogram quality have not yet been investigated thoroughly, either theoretically or empirically. However, the reconstructions of our "head" phantom were made under conditions which realistically simulated the clinical situation both in actual counting rates and image acquisition times. It appears that quantum noise does not pose too severe a problem.

Rigorous clinical trials of this and related reconstruction methods will require the development of an accurate patient-positioning device. Patient positioning is a difficult problem. The positioning apparatus used must allow the detector to be placed optimally in relation to the area of interest for each rotation view while preventing any wobble about the axis of rotation and limiting patient movement and internal organ shifting during the changes of position occurring between views. Lacking such apparatus at present, we have chosen not to include actual patient images in the present study, thinking that it would be impossible to separate the effects of poor patient positioning from problems intrinsic to the reconstruction technique and the detector system used.

The present implementation of the FTT requires a large computer and requires about 20 sec to reconstruct a single tomogram. Although this is acceptably short, the process of transferring the masses of data involved in even a single study are complex and time consuming and such processing would be expensive for large numbers of pictures. In this context it should be noted that the alternative Fourier algorithm proposed by Bracewell, et al (*12*) and discussed above can be much less demanding computationally than the FTT. We have described an implementation of this algorithm which functions with a minicomputer incorporated in a commercially available image processing system and produces acceptable tomograms with short (about 1 min/section) processing times (*7*). The implementation of this same algorithm described by Chesler (*13*) requires similar processing times, also on a minicomputer.

Our work to date has demonstrated the feasibility of applying the FTT to the imaging of radioactive objects using presently available imaging devices. More work is needed to resolve the problem of quantitative error introduced by internal absorption. However, the results obtained suggest that this could be a useful clinical tool even at the present level of development. Implementation at the clinical level will, however, require the development of adequate patient-positioning hardware.

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