METHODS FOR CALCULATING THE DEADTIME OF ANGER CAMERA SYSTEMS

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Three methods for calculating the deadtime in scintillation cameras by the two-source method are presented. Their accuracy is compared with that of seven methods that have been described.

Rapid dynamic quantitative studies with the Anger camera and multimillicurie injection of short-lived radionuclides may produce counting rates ranging from 15,000 to 30,000 cps. Even at such rapid rates, the statistics of histogram data points from small flagged areas of interest may leave much to be desired. For this reason the dose injected may be limited by the expected deadtime losses of the imaging system. At high counting rates these losses are very significant and require correction (*1*) because of the 4–8 μsec deadtime of typical Anger cameras. The addition of a computer-controlled data processing system may further extend the deadtime to 10 or 15 μsec. For a system with a deadtime of 10 μsec, a measured counting rate of 30,000 cps must be corrected by a factor of 1.43 to 42,900. An error in this deadtime of ± 1 μsec would affect the corrected value by $\pm 4.3\%$. In most systems the deadtime varies somewhat with counting rates, and before such a system is used for quantitative studies the deadtime should be measured accurately over a wide range of counting rates.

For the determination of deadtime, the familiar two-source method is both rapid and convenient (*2*). In this procedure two approximately equal sources are counted separately and in combination. However, several equations that appear in the literature for the calculation of deadtime by the two-source method (*3–11*) are approximations that assume $RT << 1$ or the background to be negligible. When using one of them for Anger camera imaging systems, in which the product, RT, of the counting rate and the deadtime may be in the range 0.2–0.4, one must be sure that the error is acceptable. The purpose of this communication is to present two precise methods of calculation developed by the authors and to compare them with a number of other equations from the literature. We also present an improved modification of one of these other equations.

The following symbols are used:

R_1, R_2, R_{12}, and Bk are measured rates from Nos. 1 and 2 sources, from the combination of the two, and the background, respectively.

N_1, N_2, and N_{12} are corresponding "true" rates corrected for deadtime.

Bc is the background rate corrected for deadtime.

T is the general symbol for deadtime.

T_1, . . ., T_{10} are values of the deadtime as calculated by a number of methods to be described.

For the two-source method:

$$N_{12} + Bc = N_1 + N_2 \text{ or}$$

$$\frac{R_{12}}{1\text{-}R_{12}T} + \frac{Bk}{1\text{-}Bk\,T} = \frac{R_1}{1\text{-}R_1T} + \frac{R_2}{1\text{-}R_2T}.$$

The background appears only on the left side of the equation because the background rate is included but once when measuring R_{12} and twice when measuring R_1 and R_2 (*7*).

Method 1. The authors have written a computer program to find a value of T_1 by successive approximation to satisfy the foregoing equation. It calculates T_1 to a precision of less than 0.01 μsec and is valid at any measured counting rate and background. For verification, the program also ouputs (N_{12} + Bc) and (N_1 + N_2) in addition to the final value for T_1.

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Method 2. The authors have also expanded the foregoing equation into a quadratic:

$$AT^2 + BT + C = 0$$

where $A = R_1R_2R_{12} + R_1R_2Bk - R_1R_{12}Bk - R_2R_{12}Bk$

$B = 2(R_{12}Bk - R_1R_2)$

$C = R_1 + R_2 - R_{12} - Bk.$

Of the two possible solutions, the only valid one is:

$$T = \frac{-B - \sqrt{B^2 - 4AC}}{2A}.$$

The authors have written a brief computer program for the calculation of this quadratic. Values agree with Method 1 to a precision of 0.002 μsec (6-digit accuracy). Slightly different roundoff or truncation errors may occur with other computers.

Methods 3–9 are taken from the literature:

Method 3. (*2,7*)

$$T_3 = \frac{\Delta}{2R_1R_2} + \frac{1}{8(R_{12} + 2Bk)}(\Delta R_{12}/R_1R_2)^2$$

where $\Delta = R_1 + R_2 - R_{12} - Bk.$

Method 4. (*7*)

$$T_4 = \frac{2(R_1 + R_2 - R_{12})}{(R_1 + R_2)R_{12}}.$$

Method 5. (*3,7,10,12,13*)

$$T_5 = \frac{R_1 + R_2 - R_{12}}{2R_1R_2}.$$

Method 6. (*4,7*)

$$T_6 = D\left[1 + \frac{D}{2}(R_{12} - 3Bk)\right]$$

where $$D = \frac{R_1 + R_2 - R_{12} - Bk}{2(R_1 - Bk)(R_2 - Bk)}.$$

Method 7. (*3*)

$$T_7 = \frac{R_1 + R_2 - R_{12}}{R_{12}^2 - R_1^2 - R_2^2}.$$

Method 8. (*3–6,11*)

$$T_8 = \frac{R_1 + R_2 - R_{12} - Bk}{R_{12}^2 - R_1^2 - R_2^2}.$$

Method 9. (*10–12*)

$$T_9 = \frac{(R_1 - Bk) + (R_2 - Bk) - (R_{12} - Bk)}{2(R_1 - Bk)(R_2 - Bk)}.$$

Review of preliminary hardcopy curves showed that of the seven methods from the literature, Method 4 provides the best precision over a wide range of factors. Only at poor signal-to-background ratios or if R_1 and R_1 differ considerably from each other, values will show a significant error. We have made an empirical background correction to Method 4 to eliminate most of the background error:

Method 10.

$$T_{10} = \frac{2(R_1 + R_2 - R_{12} - Bk)}{(R_1 + R_2 - 2Bk)(R_{12} - Bk)}.$$

Both Methods 4 and 10 require R_1 and R_2 to be approximately equal within $\pm 10\%$.

The relations between the variables N_1, Bc, T, and T_i ($i = 1, \ldots, 10$) are shown in Fig. 1 and Table 1. For these purposes the true counting rates are:

$$N_2 = N_1 \text{ and } N_{12} = 2N_1$$

A short FORTRAN program was written to display, under operator control, a wide range of these graphs on a CRT. Subsequently, a hardcopy is then obtained. For a given Bc, true deadtime, T, and counting rate N_1 this program first calculates corresponding values for R_1, R_2, and R_{12}, and Bk. It

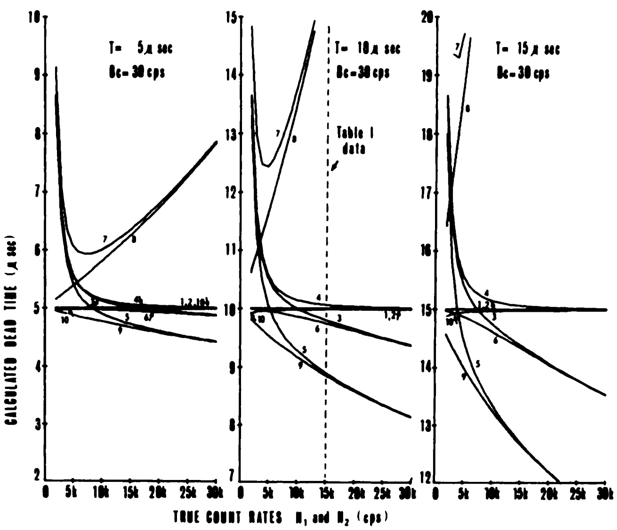

FIG. 1. Hardcopy display of results of calculating deadtime, T_i, by each of ten methods. Background is 30 cps. T_i is shown as function of corrected sample counting rate, N_1, from 2,000 to 30,000 cps, for true deadtimes of 5, 10, and 15 μsec.

TABLE 1. VALUES OF THE DEADTIME, T_i, AS CALCULATED BY EACH OF TEN METHODS

Parameters	Calculated deadtime (μsec)
$T = 10$ μsec	$T_1 = 9.995$
$N_1 = 15{,}000$ cps	$T_2 = 9.997$
$N_2 = 15{,}000$ cps	$T_3 = 9.805$
$N_{12} = 30{,}000$ cps	$T_4 = 10.064$
$Bc = 30$ cps	$T_5 = 8.894$
$R_1 = 13{,}066$ cps	$T_6 = 9.747$
$R_2 = 13{,}066$ cps	$T_7 = 15.822$
$R_{12} = 23{,}095$ cps	$T_8 = 15.666$
$Bk = 30$ cps	$T_9 = 8.847$
	$T_{10} = 9.987$

then computes the values of the deadtime as determined by each of the nine methods. The curves thus generated show T_1 as a function of N_1.

DISCUSSION

Because the two-source method involves the measurement of a small difference between two groups of counting rates, statistical errors may often be of greater importance than the computation error inherent with a particular mathematical treatment. Statistical errors of the two-source method have been discussed by Beers (*2*).

Most of the two-source method equations were developed before computers were commonly available, as approximations either to the quadratic or to a set of simultaneous equations. They were intended for G-M tube measurements at relatively low counting rates. Care must be exercised when applying them to the scintillation camera for rapid quantitative dynamic studies. In this application, precise measurements of the deadtime should be performed for any anticipated source intensity.

A series of measurements of deadtime for our systems show significant variation of deadtime with counting rate. When these data are entered into a polynomial curve-fitting program, an empirical equation is obtained relating the deadtime to the counting rate. This equation is then incorporated into the program used to correct each point on dynamic function curves.

For scintillation camera deadtime measurement, the quadratic of Method 2 is the procedure of choice when a computer is available. This program is both shorter and faster in operation than the equally precise iterative Method 1. For computation without a computer, Method 10 provides values of the deadtime remarkably close to the quadratic if both sources are of nearly equal activity. Some of the other approximation equations may be significantly or grossly wrong.

On request, the authors will be pleased to provide a listing of the program for Methods 1 or 2 in BASIC, FOCAL, or FORTRAN.

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REFERENCES

1. Harris CC, Jones RH, Buffaloe TS, et al: Stationary-detector deadtime effects on dynamic radionuclide biologic measurements. *J Nucl Med* 11: 324, 1970

2. Beers Y: A precision method of measuring Geiger counter resolving time. *Rev Sci Instrum* 13: 72–73, 1942

3. Quimby EH, Feitelberg S: *Radioactive Isotopes in Medicine and Biology*, Philadelphia, Lea and Febiger, 1963, pp 231–232

4. Bleuler E, Goldsmith GJ: *Experimental Nucleonics*. New York, Rinehart and Co, 1958, pp 60–62

5. Price WJ: *Nuclear Radiation Detection*. New York, McGraw Hill, 1958, pp 126–127

6. Friedlander G, Kennedy JW, Miller JM: *Nuclear and Radiochemistry*, New York, John Wiley and Sons, 1964, pp 186–187

7. Chase GD, Rabinowitz JL: *Principles of Radioisotope Methodology*, 3rd ed, Minneapolis, Burgess Publishing Co, 1967, pp 114–117

8. Rainwater LJ, Wu CS: Application of probability theory to nuclear particle detection. *Nucleonics* 2: No 1, 42–43, 1948

9. Preuss LE: Constant geometry for deadtime determination. *Nucleonics* 10: No 2, 62, 1952

10. Lapp RE, Andrews HL: *Nuclear Radiation Physics*, 3rd ed, Englewood Cliffs, New Jersey, Prentice-Hall, 1963, p 358

11. Attix FH, Roesch WC: *Radiation Dosimetry Second Edition Volume II Instrumentation*. New York, Academic Press, 1966, p 100

12. Young MEJ: *Radiological Physics*. Springfield, Ill, Charles C Thomas, 1967, p 280

13. Johns HE, Cunningham JR: *The Physics of Radiology*, 2nd ed, Springfield, Ill, Charles C Thomas, 1961, p 544