REVISED PARAMETERS FOR USE IN LOEVINGER'S BETA POINT-SOURCE DOSE-DISTRIBUTION FUNCTION

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Even though extensive data in tabular form on beta point-source dose-distribution in water and air have been made available by Cross and Berger, Loevinger's beta point-source dose-distribution function is the only equation available in an analytical form that can be used to calculate the dose from extended sources. However, constants employed in this equation were based on scanty experimental data which were available in the 1950s. It was seen that Loevinger's original parameters give fairly accurate dose-distribution values for beta emitters of maximum energy less than 1 MeV, while for high-energy emitters the dose values using the original constants were less by about 20% compared with Cross's values. Hence, the parameters in Loevinger's equation have been reevaluated for high-energy beta emitters.

On the basis of experimental results Loevinger, et al (*1*) have represented the beta-ray point-source dose distribution in air and tissue in the form of an empirical equation

$$J(x) = \frac{\kappa}{(\gamma x)^2}\left\{c\left[1 - \frac{\gamma x}{c}e^{1-\left(\frac{\gamma x}{c}\right)}\right] + \gamma x e^{1-\gamma x}\right\} \quad (1)$$

where

$$\left[1 - \frac{\gamma x}{c}e^{1-\left(\frac{\gamma x}{c}\right)}\right] \equiv 0 \text{ when } x \geqq c/\gamma,$$

$$\kappa = 1.28 \times 10^{-9}\, P^2\, \gamma^3\, \bar{E}_\beta\, \alpha \text{ rad per disintegration} \quad (2)$$

$$\alpha = [3c^2 - (c^2 - 1)e]^{-1} \quad (3)$$

$$\gamma = \frac{18.6}{(E_0 - 0.036)^{1.37}}\left(2 - \frac{\bar{E}_\beta}{\bar{E}_\beta^*}\right) \text{cm}^2/\text{gm of tissue} \quad (4)$$

$$c = \left.\begin{cases} 2 & \text{for } 0.17 < E_0 < 0.5 \text{ MeV} \\ 1.5 & \text{for } 0.5 \leqq E_0 < 1.5 \text{ MeV} \\ 1 & \text{for } 1.5 \leqq E_0 < 3 \text{ MeV} \end{cases}\right\} \text{for soft tissue} \quad (5)$$

$J(x)$ is the dose in rads per disintegration at a distance x from a point source of beta radiation and γ is the apparent absorption coefficient. The product γx is dimensionless. $\bar{E}_\beta$ is the average beta-ray energy per disintegration and $\bar{E}_\beta^*$ the average beta-ray energy per disintegration for a hypothetical allowed spectrum if the decay is forbidden, with the same maximum beta-ray end-point energy E_0. For allowed spectra $\bar{E}_\beta/\bar{E}_\beta^*$ is unity. P is the density in gm/cm^3 of the medium.

The function $J(x)$ is empirical, and the constants are based on scanty experimental data that were available in the 1950s. Loevinger has suggested the reevaluation of the constants when more data became available (*2*).

Spencer (*3*) has calculated the dose distributions in various media from point sources of monoenergetic electrons. Cross has used these data to tabulate the dose distributions from point beta emitters in

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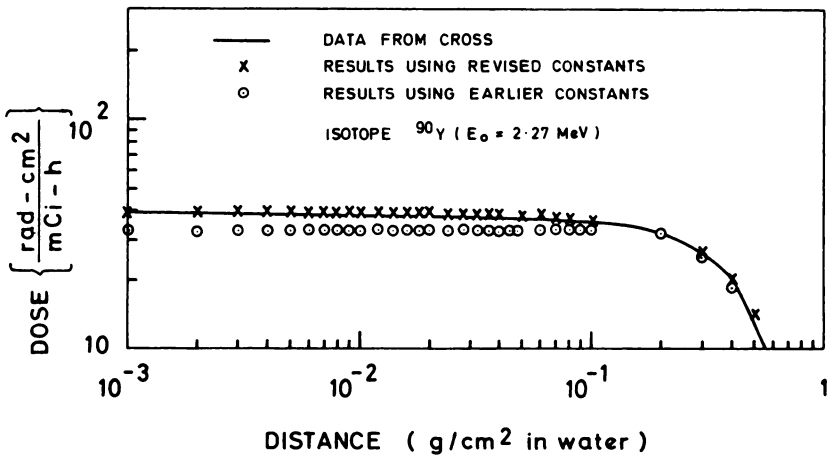

FIG. 1. Dose distribution for ^{90}Y in water.

water (*4*). He has also measured dose distribution (*5*) around point beta sources and found agreement within ±4% with the tabulated values. Berger (*6*) has also presented similar tables which are in agreement with Cross's tables.

PRESENT WORK

While these tables are very useful, an analytical representation in a form similar to that given by Loevinger's equation would be useful in calculating the dose distribution from a beta emitter uniformly distributed in a medium. Therefore an investigation was undertaken to see how far the dose distributions in water as given by Eq. 1 with the constants evaluated according to Eqs. 2–5 agree with the tabulated data of Cross. It was found that, in general, for low-energy beta emitters ($E_0 \leq 1$ MeV) with simple decay schemes dose distributions evaluated in this way agree reasonably well with the tables of Cross.

For higher energies, however, Loevinger's equa-

TABLE 1. REVISED PARAMETERS FOR WATER AND AIR FOR USE IN LOEVINGER'S EQUATION

		Water			Air		
Isotope	E_0 (MeV)	γ (cm^2/gm)	C	κ(rad/dis-integration)	γ (cm^2/gm)	C	κ(rad/dis-integration)
^{14}C	0.16	346.3	2.20	6.43×10^{-4}	315.8	1.90	8.89×10^{-10}
^{35}S	0.17	323.8	2.35	5.24×10^{-4}	272.0	2.17	5.19×10^{-10}
^{203}Hg	0.208	219.9	2.19	1.94×10^{-4}	192.2	2.28	2.11×10^{-10}
^{45}Ca	0.26	156.7	2.01	9.89×10^{-5}	138.5	2.05	1.14×10^{-10}
^{59}Fe	0.48(51%) 0.27(48%)	64.3	2.58	8.78×10^{-6}	236.5	3.95	4.71×10^{-10}
^{90}Sr	0.54	41.2	1.14	5.71×10^{-6}	35.7	1.74	6.14×10^{-12}
^{64}Cu	0.57	44.2	1.60	2.42×10^{-6}	39.2	2.04	2.73×10^{-12}
^{131}I	0.61(87%) 0.33(9.3%) 0.25(2.8%)	41.1	2.67	4.52×10^{-6}	35.4	1.93	4.63×10^{-12}
^{198}Au	0.96	21.1	1.46	1.15×10^{-6}	19.4	1.38	1.48×10^{-12}
^{132}I	1.22(24%) 1.71(21%) 0.80(21%) 1.04(15%) 2.16(18%)	10.6	2.44	1.90×10^{-7}	9.2	2.69	1.93×10^{-12}
^{24}Na	1.39	11.8	1.15	3.95×10^{-7}	10.5	1.13	4.76×10^{-13}
^{89}Sr	1.46	10.3	1.38	2.76×10^{-7}	8.9	1.41	2.93×10^{-13}
^{32}P	1.71	8.7	1.13	2.04×10^{-7}	8.3	1.02	2.84×10^{-13}
^{90}Y	2.27	5.7	1.26	7.51×10^{-8}	5.2	1.19	1.02×10^{-13}
^{144}Pr	2.98	4.51	1.14	4.96×10^{-8}	3.9	1.13	5.70×10^{-14}
^{106}Rh	3.53(68%) 3.11(11%) 2.44(12%) 2.00(3%)	4.1	1.06	4.20×10^{-8}	3.6	1.06	4.73×10^{-14}
^{42}K	3.6(82%) 2.0(18%)	3.7	1.16	3.1×10^{-8}	3.3	1.13	3.71×10^{-14}

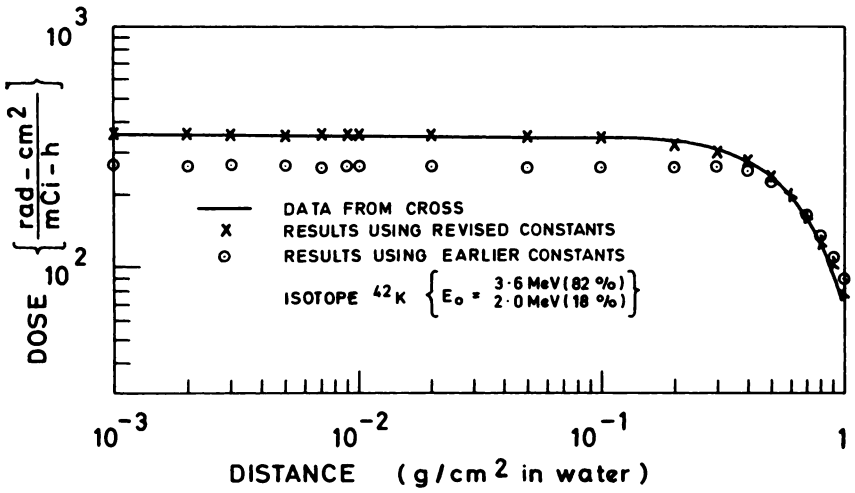

FIG. 2. Dose distribution for ^{42}K in water.

tion, evaluated as outlined above, gave dose values which were about 20% lower than those in the tables. It was therefore decided to reevaluate the constants used in Eq. 1 to get better agreement with the tables.

This was done graphically as follows. For distances greater than c/γ,

$$xJ(x) = \frac{\kappa}{\gamma} e^{1-\gamma x} \quad (6)$$

$$\ln xJ(x) = -\gamma x + (1 + \ln \kappa/\gamma). \quad (7)$$

A plot of $\ln xJ(x)$ against x gave a straight line with a slope and an intercept $[1 + (\ln \kappa/\gamma)]$. Using $J(x)$ values derived from Cross's tables, revised values of γ and κ/γ were obtained. The constant c was evaluated by using the relationship

$$\lim_{x \to 0} x^2 J(x) = \frac{\kappa}{\gamma^2} c. \quad (8)$$

Table 1 shows the revised constants obtained in this fashion for some of the isotopes that have important medical applications. These revised constants when used in Eq. 1 give results in agreement with the tabulated data of Cross. Figures 1–3 compare the dose distribution in water which was obtained by using the earlier constants as well as the revised constants. The ordinate in all these figures is rad-cm²/mCi/hr which is

$$= 1.332 \times 10^{11} x^2 J(x)$$

where 1.332×10^{11} is the disintegration rate of 1 mCi in 1 hr.

It was seen that the absorption coefficients obtained from Eq. 4 and those derived by the graphical method agree well over the entire region of energies studied. Differences have been observed between the old and new κ and c values. Figure 4 is a plot of the revised values of γ against E_0 for some of the

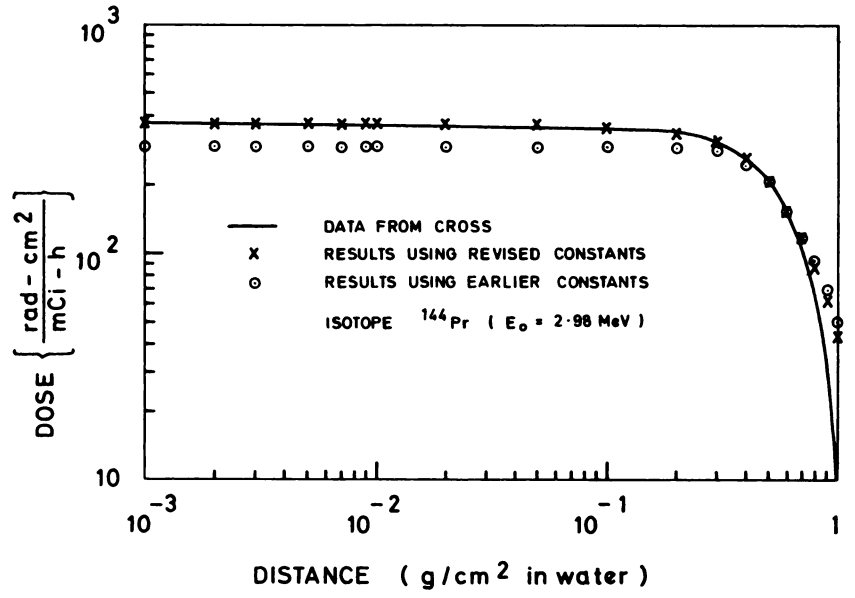

FIG. 3. Dose distribution for ^{144}Pr in water.

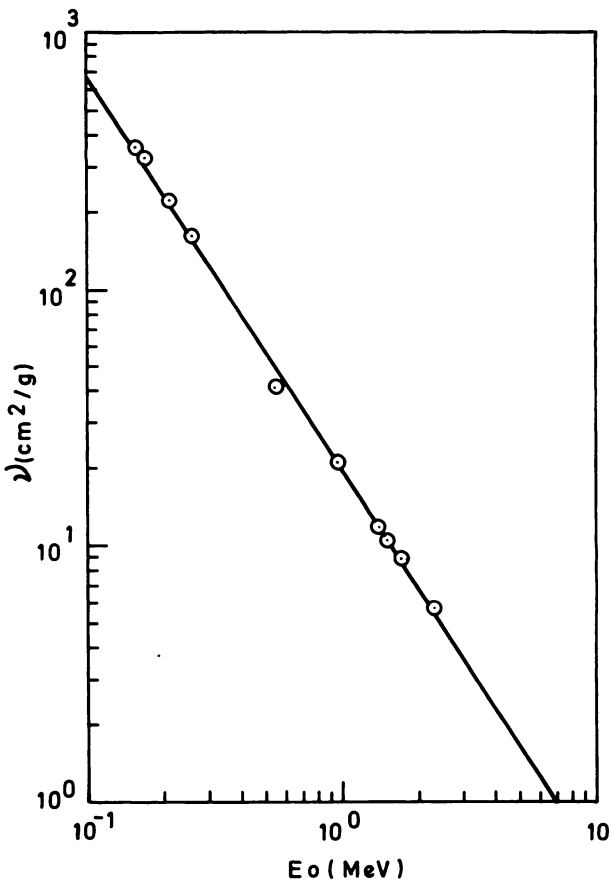

FIG. 4. Variation of absorption coefficient with energy.

isotopes having simple decay schemes. It was found that the absorption coefficient can be expressed as

$$\gamma = 19.97\ E_0^{-1.537}\ \text{cm}^2/\text{gm}$$

which gives γ values within $\pm 2\%$.

The method was extended to air and the values of the revised constants for air are also presented in Table 1.

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