# FRESNEL ZONE PLATE IMAGING IN NUCLEAR MEDICINE

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Considerable progress has been made in recent years in detecting the scintillation pattern produced by a gamma-ray image. Systems such as the Anger camera (*1*) and Autoflouroscope (*2*) give efficient counting while an image intensifier camera (*3,4*) gives better spatial resolution at some sacrifice in efficiency. However, the common means of image formation, the pinhole aperture and parallel-hole collimator, are very inefficient. Only a tiny fraction ($\sim$0.1–0.01%) of the gamma-ray photons emitted by the source are transmitted to the detector plane (scintillator crystal), and this fraction can be increased only by unacceptably degrading the spatial resolution. It would be desirable, of course, to have a large-aperture, gamma-ray lens so that good collection efficiency and good resolution could be obtained simultaneously. Strictly speaking, a gamma-ray lens is impossible because gamma rays are not appreciably refracted by matter. However, this paper describes a "coded" aperture which performs the same function and permits an increase in collection efficiency of two to three orders of magnitude over a pinhole or parallel-hole collimator. It was first proposed by Mertz and Young (*5,6*) for use in x-ray astronomy.

The basic concept is illustrated in Fig. 1. The aperture is a Fresnel zone plate (*7,8*), thin enough so that there is essentially no collimation. The zone plate has a series of equi-area annular zones, alternately transparent and opaque to the gamma rays, with the edges of the zones located at radii given by

$$r_n = r_1 \sqrt{n}; \qquad n = 1,2,\ldots\ldots N. \qquad (1)$$

To understand the operation of this aperture, consider first a point source of gamma rays. Then the scintillation pattern on the crystal is a projected shadow of the zone plate, with the position of the shadow depending linearly on the position of the source. The shadow thus contains the desired information about the source location. It may be regarded as a coded image similar to a hologram. Indeed, a Fresnel zone plate is simply the hologram of a point source (*9*). The essential difference from normal holography is that here the "hologram" was formed by shadowing rather than by diffraction.

Decoding or reconstruction is accomplished with a laser beam as in optical holography. The scintillation pattern on the crystal is detected, in our experiments with an Anger camera, and the output display is photographed as a positive transparency. This transparency is then the optical hologram corresponding to the coded gamma-ray image.

When a coherent light beam passes through the transparency, some of the light is diffracted by the zone plate into a small focal spot which is the image of the original gamma-ray point source. If two point sources were present, two distinct focal points would be produced. Then, to the extent that linear superposition holds, any general object may be considered to be made up of point sources and will be correctly imaged.

The major practical problem is the undiffracted or DC light which results from the incoherent nature

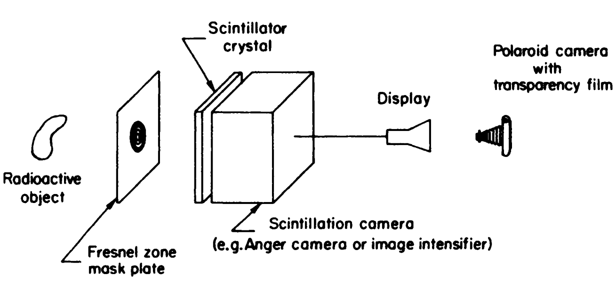

**FIG. 1.** **Fresnel zone plate camera. Zone plate is heavy metal such as lead mounted on backing plate of light metal such as aluminum and machined into zones which are alternately transparent and opaque to radiation.**

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of the gamma radiation and the fact that a shadow is recorded rather than a diffraction pattern. In optical holography the diffraction amplitude recorded in the hologram has both positive and negative values with a zero average value. The gamma-ray flux plays the role of the amplitude in the present case, but, of course, cannot be negative. Much of the difficulty with the DC light is eliminated by placing a Schlieren stop in the focal plane of a lens as shown in Fig. 2. The optical amplitude distribution in this plane is the Fourier transform of the hologram transparency (*8*). Therefore the DC light is concentrated near the axis and is blocked by the stop. However, some of the desired information in the hologram also contains low spatial frequencies which are blocked by the stop, particularly for large objects. In addition, there are fluctuations associated with the DC transparency of the zone plate which cannot be blocked and which contribute additional noise to the reconstruction. The net result is that small objects are more easily imaged than large ones.

The characteristics of this imaging system are derivable from the well-known focusing properties of a zone plate (*7,8*). A detailed treatment will be published separately. The minimum resolvable (Rayleigh criterion) separation of two point sources, $\delta$, is given by

$$\delta = \beta(N)\, \Delta r_N \frac{S_1 + S_2}{S_2}, \tag{2}$$

in which $\beta(N)$ is a parameter of order unity which depends on the number of rings in the zone plate (*10*), $S_1$ is the distance from object to zone plate, and $S_2$ is the distance from zone plate to camera. $\Delta r_N$ is the width of the narrowest (outermost) zone in the plate, i.e.,

$$\Delta r_N \equiv r_N - r_{N-1} \approx r_1/2(N)^{1/2} = r_N/2N. \tag{3}$$

The scintillation camera should be capable of resolving this outer ring since an unresolved ring contributes noise but no image information. Therefore the camera should satisfy the condition

$$\Delta r_N \frac{S_1 + S_2}{S_1} \geq d_{cam} \tag{4}$$

in which $d_{cam}$ is the intrinsic camera resolution. Equation 2 shows that the zone plate aperture has a resolution equivalent to a pinhole aperture where the pinhole diameter is equal to $\beta(N)\Delta r_N$. The final image resolution with a zone plate camera may be somewhat degraded by film nonlinearities and lens aberrations in the reconstruction system, but our experience has been that the theoretical limiting resolution, Eq. 2, can be closely approached in practice.

The other properties of the zone plate camera relate to the diameter of the scintillator crystal, $D_{xtal}$. Usually it is desirable to have a point source project the zone plate so that it about fills the crystal:

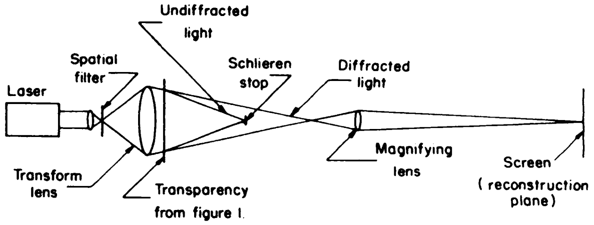

**FIG. 2.** Reconstruction system for decoding image obtained with zone plate camera.

$$2r_N \frac{S_1 + S_2}{S_1} \approx D_{xtal}. \tag{5}$$

If $r_N$ is much smaller than this, collection efficiency is sacrificed. If it is much larger, resolution must be sacrificed if Eq. 4 is to be satisfied.

Equation 5 implies that the collection aperture varies with position in the field. We may define a field of view (FOV) as the diameter of a circle in the object plane for which the center of the zone plate projects to the edge of the crystal. The efficiency at the edge of this field will be somewhat less than one half of its value at the center. Then by simple geometry,

$$FOV = D_{xtal} \frac{S_1}{S_2} \tag{6}$$

which is the same as for a pinhole camera except there the falloff in efficiency at the edge of the field is abrupt while for the zone plate camera it is gradual. Furthermore the resolution also varies over the field of a zone plate camera, being only about one half as good at the edge defined by Eq. 6 as at the center. If flatness of efficiency and resolution over the field is important, it can be achieved at the expense of center efficiency and resolution by reducing $r_N$ below the value given in Eq. 5, or at the expense of center resolution alone by increasing $r_N$. As with a pinhole camera, very small objects can be well resolved by making $S_1/S_2$ or $\Delta r_N$ small enough, but the price must be paid in FOV or efficiency.

Note that Eqs. 4 and 5 fix the maximum number of rings (opaque plus transparent) in the zone plate, since

$$\frac{\Delta r_N}{2r_N} = \frac{1}{4N} \geq \frac{d_{cam}}{D_{xtal}}, \tag{7}$$

independent of $S_1/S_2$. It is desirable to make N as large as possible for maximum resolution. Therefore we should have

$$N \approx \frac{D_{xtal}}{4d_{cam}} \tag{8}$$

which for an Anger camera means that $N \approx 6$–$7$.

**TABLE 1. CALCULATED PARAMETERS OF FRESNEL ZONE PLATE CAMERAS**

| | | Zone plate parameters | | Imaging properties | | | |
|---|---|---|---|---|---|---|---|
| Assumed detector parameters | Spacing ratio ($S_1/S_2$) | No. of zones (N) | Smallest width ($\Delta r_N$) (mm) | Resolution ($\delta$) (mm) | FOV (cm) | Depth of field ($\Delta S_1/S_1$) (%) | Relative efficiency ($\eta_{zp}/\eta_{ph}$) |
| Anger camera | ½ | 6 | 3.3 | 6.4 | 12 | 9.3 | 170 |
| $d_{cam} = 1$ cm | 1 | 6 | 5.0 | 13 | 24 | 12.4 | 170 |
| $D_{xtal} = 24$ cm | 2 | 6 | 6.7 | 29 | 48 | 18.6 | 170 |
| Image intensifier | ½ | 25 | 0.67 | 1.2 | 10 | 2.25 | 3300 |
| $d_{cam} = 2$ mm | 1 | 25 | 1.0 | 2.4 | 20 | 3.0 | 3300 |
| $D_{xtal} = 20$ cm | 2 | 25 | 1.33 | 4.9 | 40 | 4.5 | 3300 |
| Image intensifier | ½ | 67* | 0.67 | 3.8 | 10 | 6.75 | 367 |
| $d_{cam} = 2$ mm | | | | | | | |
| $D_{xtal} = 20$ cm | 1 | 67 | 1.0 | 7.2 | 20 | 9 | 367 |
| (with off-axis | | | | | | | |
| zone plate) | 2 | 67 | 1.33 | 15 | 40 | 13.5 | 367 |

* Off-axis zone plate shifted by one diameter, including zones 8 through 75.

The gamma-ray collection efficiency $\eta_{zp}$ of the zone plate camera relative to a pinhole of the same resolution and same spacings $S_1$ and $S_2$, is

$$\frac{\eta_{zp}}{\eta_{ph}} = \frac{\text{transparent area of zone plate}}{\text{area of equivalent pinhole}}$$

$$= \frac{\frac{1}{2}(2r_N)^2}{(\beta \Delta r_N)^2} = \frac{8N^2}{\beta^2}. \qquad (9)$$

For $N = 7$ this ratio is 230, which means that for a point source, an image with a given signal-to-noise ratio can be obtained 230 times faster with a zone plate camera than with a pinhole. For more complicated objects the advantage is less since more counts are required to get a comparable image quality in the zone plate case. Exactly how many more is still to be resolved. Experimental and theoretical investigation of this problem is underway, but results to date are inconclusive.

Another advantage of the zone plate camera is that three-dimensional information about the object is recorded in a single exposure on a single hologram, just as in optical holography. For the zone plate camera, the reconstruction is tomographic—only a thin slice of the object is in focus at once. Different planes can be brought into focus simply by changing focal distances in the optical processing system. The depth of field, defined as the range in $S_1$ which an object must move for a two-fold decrease in resolution, is given by

$$\frac{\Delta S_1}{S_1} \approx \frac{3}{8N} \frac{S_1 + S_2}{S_2}. \qquad (10)$$

As the numerical examples in Table 1 show, this *fractional* depth of field is usually quite small. However, if an extended *absolute* depth of field is desired, rather than a tomographic display, it can be obtained at the expense of sensitivity by increasing $S_1$ and $S_2$ in the same proportion.

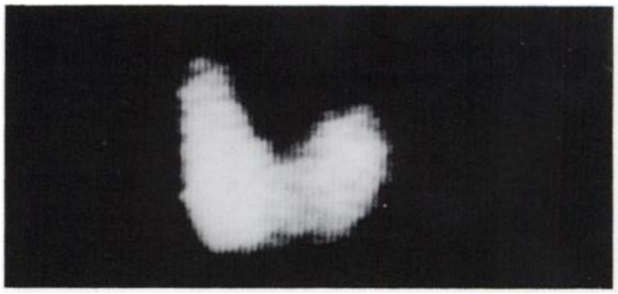

**FIG. 3.** Reconstructed image of 10-$\mu$Ci thyroid phantom taken with lead zone plate (on-axis) and Anger camera.

A reconstruction of a 10 $\mu$Ci simulated $^{131}$I thyroid phantom is shown in Fig. 3. A total of 110,000 counts were collected by the Anger camera* in a 3-min exposure with $S_1 = 30$ cm and $S_2 = 15$ cm. The counting rate could be increased about an order of magnitude, if this were important, simply by using $S_1 = 10$ cm and $S_2 = 5$ cm. Further reduction in these values would result in a loss in resolution due to the finite crystal thickness.

The phantom has two cold nodules, one of which shows up as a shortened right lobe. The other is not detected because of inadequate resolution. The theoretical resolution $\delta$ is about 2 cm, and inspection of Fig. 3 shows that the actual value is not substantially worse.

Figure 3 also indicates that 110,000 counts are completely adequate for this object with the zone plate aperture, and probably fewer could have been used. A pinhole image of the same object required about 15,000 counts for comparable image quality.

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* Nuclear-Chicago Pho/Gamma III, without high performance modifications.

A useful variation on the system described above is an off-axis zone plate to aid in separating the DC and information-bearing components of the light beam (*11*). This approach, which is analogous to using an off-axis reference beam in optical holography, gives much cleaner images but poorer object resolution for a given camera resolution*. Therefore it is much more suitable for image-intensifier cameras than for an Anger camera. Comparative performance figures are given in Table 1.

The Fresnel zone plate aperture is but one member of a large class of possible coding functions which can be used to image incoherent radiation. Dicke (*12*) has proposed a random array of pinholes for the same purpose. Also, the author (*13*) has described the use of a linear array of pinholes with varying spacing, or essentially a one-dimensional off-axis zone plate. Here the decoding was done electrically with a dispersive filter rather than optically. An excellent discussion of the requirements on the codes is given by Mertz (*11*).

## SUMMARY

In summary, we have shown that a Fresnel zone plate aperture permits the recording of a gamma-ray image in a coded form, analogous to a hologram, but with an enormous increase in gamma-ray collection efficiency compared to a pinhole or collimator. This advantage can be used by the physician to either reduce the administered dose of radiopharmaceuticals or to decrease the time required to obtain an image. The practicality of the scheme has been demonstrated for small objects such as the thyroid gland. For high-resolution imaging of large objects, such as the liver, there are definite limitations which we will delineate more carefully in forthcoming publications. Future research in this field should include the investigation of other possible aperture codes and associated decoding methods, extension to other types of radiation, and most importantly, clinical evaluation.

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* The resolution, field of view, and depth of field of a zone plate camera using an off-axis zone plate are the same as those of a camera using an equivalent on-axis plate having the same diameter and focal length. However, the off-axis plate will have a minimum zone width $\Delta r_N$ which is smaller by a factor of 2 to 3 (depending on how far off-axis the plate is) and therefore requires a smaller $d_{cam}$. The equations given in the text hold only for an on-axis zone plate.