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THE INDEX OF RESOLUTION WHEN SEPTAL PENETRATION IS IMPORTANT

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At present, it is recommended that the resolving power of collimators be described in terms of the modulation transfer function (MTF) (*1*). The MTF is, however, an abstract concept not easily understood by those unfamiliar with Fourier analysis. Thus many authors prefer to quote the resolving power in terms of the index of resolution which is the full width at half maximum of the linespread function (FWHM). As an example, the work of Tsialas and Hine is cited (*2*). When septal and edge penetration is of importance, the FWHM does not carry all the information necessary to characterize the collimator (*3,4*). In this note, an attempt is made to remove this inadequacy.

Consider a focused collimator scanning over a line source of radioactivity as illustrated in Fig. 1. The recorded counting rate as a function of position in the scan direction is illustrated on the right. It is commonly known as the linespread function, and according to Tsialas and Hine (*4*) it should be fitted by a sum of three Gaussians. For our analysis we will assume that the linespread function can be approximated by two Gaussians or

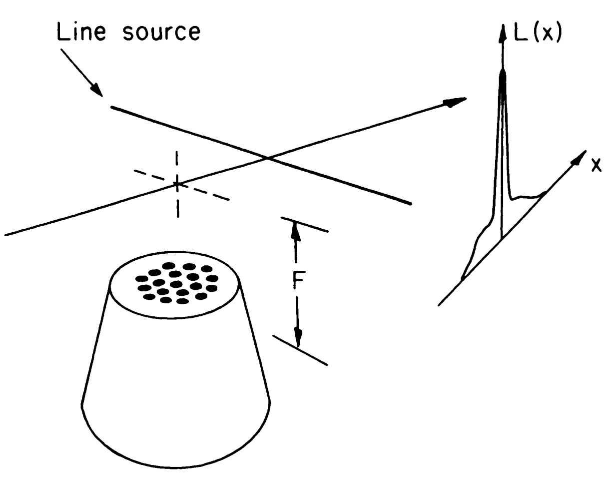

FIG. 1. Measurement of line-spread function.

$$L(x) = \frac{A_1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{x^2}{2\sigma_1^2}\right] + \frac{A_2}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{x^2}{2\sigma_2^2}\right] \qquad (1)$$

where x is distance in the scan direction, A_1 and A_2 are constants that depend on the collimator geometry, A_2 also depends on the gamma-ray energy, and σ_1 and σ_2 are constants that depend on the channel and collimator geometry and on the distance between the face of the collimator and the plane containing the radioactivity.

Although Eq. 1 is a fit to experimental data, there is theoretical justification for choosing such a fit. Thus it was shown in a previous publication (*5*) that in the absence of septal penetration, the rigorous expression for the linespread function can be approximated by a Gaussian by virtue of the central limit theorem. We may thus identify the first Gaussian with that fraction of radiation transmitted by the channel and useful in the making of the image. If the source of radioactivity is in the focal plane, then

$$A_1 = \frac{\pi a^2 c^2 N}{4L^2} \qquad (2)$$
$$\sigma_1^2 = \left[\frac{F}{L}\,a\right]^2$$

where a is the radius of the exit pupil of a single channel, c is the radius of the entrance pupil of a single channel, L is the length of the collimator, F is the focal length, and N is the number of channel.

The second Gaussian is then viewed as that fraction of radiation that reaches the detector through septal or edge penetration. It can be considered as noise and is in part responsible for a deteriorated image. Unlike the first Gaussian, there is no theo-

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retical justification for choosing such a fit. Only the experimental data in the literature suggests the approximation. However, beginning with Mather (*6*), many investigators have attempted and succeeded at studying quantitatively the problem of septal and edge penetration. As a recent reference, the work of Simons (*7*) is cited. There is also much experimental data in the literature similar to the already cited work of Tsialas and Hine (*2*), the most comprehensive study being perhaps the work of Harris et al at Oak Ridge (*8*). These data suggest that σ_2 is a function of the radius of the circle that surrounds all the channels at the face of the collimator. A_2 is also a function of the energy of the gamma rays.

By the "index of resolution" one usually means the smallest distance d below which two parallel-line sources of equal intensity cannot be distinguished. This will depend on the shape of the linespread function. Also, it must be emphasized that a criterion for two line sources of equal intensity is not valid for two line sources of unequal intensity (*9*). When the linespread function has the shape of a Gaussian, then the "index of resolution" is usually identified with the FWHM because when the mean of the two curves are separated by a distance smaller than the FWHM, there is considerable overlap and the summation curve has the shape of a single Gaussian. When septal penetration is unimportant, Eq. 1 reduces to a single Gaussian, and the index of resolution d reduces to the FWHM or

$$d = 2.36\,\sigma_1. \tag{3}$$

When septal and edge penetration cannot be neglected, the index of resolution must be redefined. In parallel to statistics, it is suggested that the index of resolution be redefined in terms of the second-order central moment. In statistics the width or dispersion of a probability distribution is described by the variance which is synonymous with the second-order central moment. The difference between two distributions is evaluated in terms of the difference of the two means and the area of overlap which depends strongly on the variance. Let η be the mean or

$$\eta = \frac{\int_{-\infty}^{+\infty} xL(x)dx}{\int_{-\infty}^{+\infty} L(x)dx}. \tag{4}$$

The second central moment is defined as

$$\overline{(x-\eta)^2} = \frac{\int_{-\infty}^{+\infty} (x-\eta)^2 L(x)dx}{\int_{-\infty}^{+\infty} L(x)dx}, \tag{5}$$

or using Eq. 1 we obtain

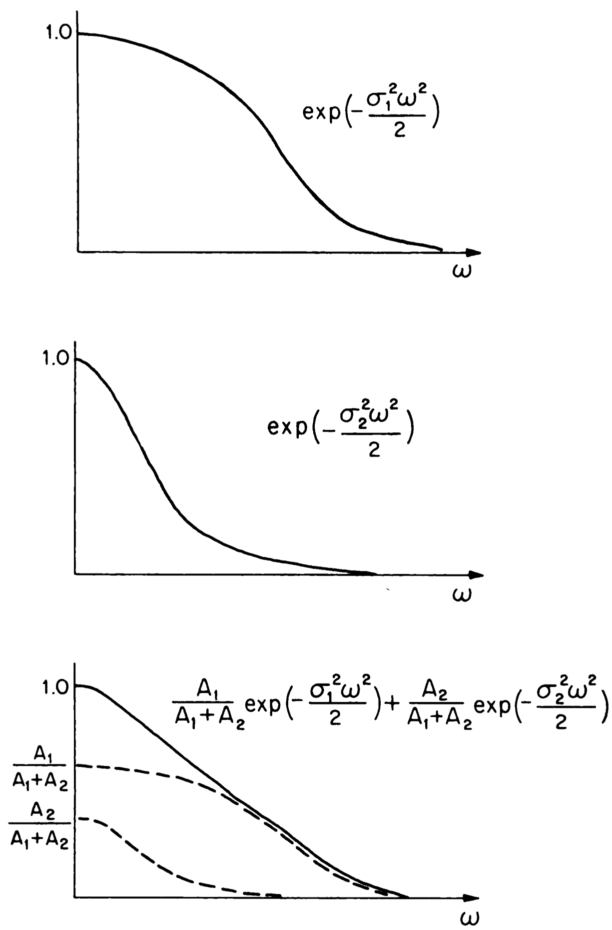

FIG. 2. Modulation transfer function when septal penetration is important.

$$\overline{(x-\eta)^2} = \frac{A_1}{A_1+A_2}\sigma_1^2 + \frac{A_2}{A_1+A_2}\sigma_1^2. \tag{6}$$

The index of resolution can now be redefined as

$$d = 2.36\sqrt{\overline{(x-\eta)^2}.} \tag{7}$$

This index of resolution has the following explanation. For low-energy gammas, there is no edge or septal penetration, $A_2 \simeq 0$, and we have an ideal collimator. At high energies, the septas have little effect in stopping the unwanted radiation, $A_2\sigma_2^2 >> A_1\sigma_1^2$, and the collimator acts as a single-hole collimator.

We next investigate what this means in terms of the MTF. Mathematically, the modulation transfer function is the mapping of the linespread function into frequency space. It is obtained by taking the Fourier transform of the linespread function and normalizing it to 1.00 at frequency zero. For the linespread function described by Eq. 1 this yields

$$M(\omega) = \frac{A_1}{A_1 + A_2} \exp\left[-\frac{\sigma_1^2\omega^2}{2} \right] + \frac{A_2}{A_1 + A_2} \exp\left[-\frac{\sigma_2^2\omega^2}{2} \right]. \tag{8}$$

If penetration effects are negligible, then the modulation transfer function has a shape as illustrated at the top of Fig. 2. The shape of that part of the transfer function accounting for penetration effect is illustrated in the middle of Fig. 2. When penetration effects are important, the resulting transfer function is as illustrated at the bottom of Fig. 2. Clearly, it is inferior to the one at the top of the figure. The higher spatial frequencies have been attenuated at the expense of the lower ones.

Whether the index of resolution or the MTF is the more appropriate quality factor to describe collimator performance depends on the problem that one seeks to solve or describe.

An example of where the index of resolution is more useful is the imaging of line-test pattern. Here a quotation of the index of resolution allows a brief description.

An example of where the modulation transfer function is very powerful is the design of a circuit that removes the distortion introduced by septal penetration. This is illustrated in Fig. 3. A duplicate of the image is passed through a low band pass filter. The resulting image is attenuated by $A_2/A_1 + A_2$ and subtracted from the original image. In radiology such filtering is achieved by means of LogEtronics circuits (*10*). It can also be accomplished by computers. This problem could not have been described so briefly in terms of the index of resolution.

SUMMARY

The index of resolution of focused collimators is redefined in terms of the second central moment. This index of resolution depends on the energy of the gamma ray. For low-energy gamma rays, it reduces to the FWHM of the linespread function.

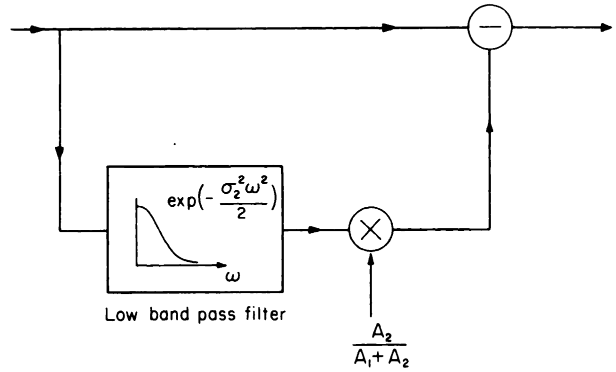

FIG. 3. Circuitry for removing image resulting from septal penetration.

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