COMPUTER PROCESSING OF SCANS USING FOURIER AND OTHER TRANSFORMATIONS

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In theory the ability of the radionuclide scanner to detect small lesions can be improved by computer processing. We begin by assuming that all of the radioactivity is distributed in the focal plane of the collimator as a two-dimensional function A(s) (Fig. 1). This simplifying assumption is necessary to bring the solution of the problem within practical limits. Ignoring fluctuations due to the random process of decay, the observed scan O(s) is seen to be the result of the convolution of the actual nuclide distribution A(s) and another function C(s), the point source response of the scanning system. Since O(s) and C(s) are known, this equation can be solved (deconvolved) for A(s). Other investigators have suggested this approach (*1–5*) and Nagai and Iinuma have described an iterative method of solving the convolution integral in the spatial domain to improve resolution (*6*). Unfortunately, the computing times necessary to carry out iterative correction are too long to be practical, and if one attempts to approach A(s) exactly, false oscillations in the processed scan are generated. A differential operator method also proposed by Nagai and Iinuma (*7*) is somewhat faster but results are too sensitive to statistical fluctuations in the scan.

An alternate means of performing deconvolution is to Fourier transform the functions from the spatial domain to the frequency domain where division of the transform of the observed function $O(\omega)$ by the transform of the point source response function $C(\omega)$ results in the Fourier transform of the actual distribution $A(\omega)$. Taking the inverse Fourier transform of $A(\omega)$ results in the spatial domain representation of the actual nuclide distribution A(s).

A 128 $\times$ 128 sampled data array is necessary to adequately describe most scans. In the past, the straightforward computation of a two-dimensional Fourier transform of an array this size required very long computation times. Recently so-called Fast Fourier Transform (FFT) computer programs have been developed which make frequency domain deconvolution practical. We have adapted a version of one of these—the multidimensional Cooley-Tukey Fast Fourier algorithm—for use in scan analysis (*8*).

In the absence of noise the deconvolution operation could be carried out exactly in the manner described above. However, an observed scan represents not only the actual nuclide distribution but also a

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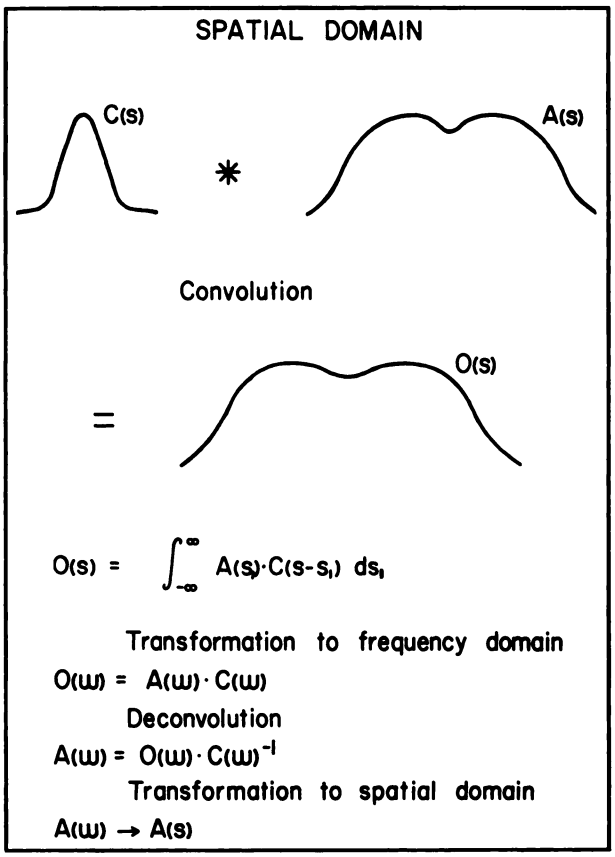

FIG. 1. Linear digital filtering principle.

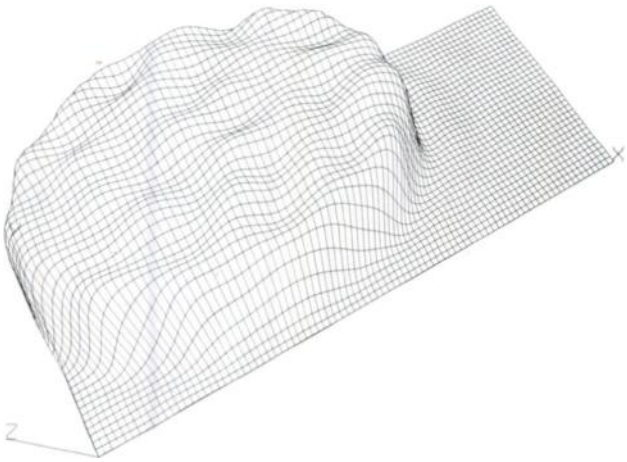

FIG. 2. Transformed and filtered scan of circular uniform disk of radioactivity using sharp cutoff filter. Ripple produces smooth undulation across surface.

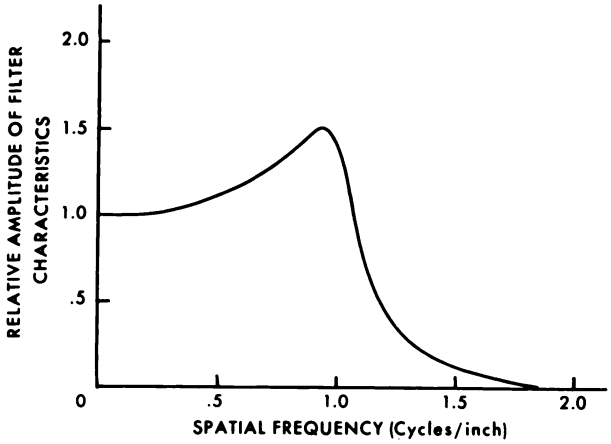

FIG. 3. Results with one of frequency filters presently in use.

considerable amount of additive "noise" due to the random nature of radionuclide decay. This noise greatly complicates practical considerations involved in correcting the scan by deconvolution. If one attempts to do a deconvolution on raw data, the result is degraded due to amplification of this noise. Some smoothing is necessary, but this can be accomplished simultaneously with deconvolution. In the frequency domain the coefficients representing higher frequencies are predominantly caused by noise and are distributed in a separate part of the frequency array. We assume that frequency components above that of the resolving power of the collimator are due to noise, and we set these high-frequency coefficients to zero. Although simple in theory, the formidable problem introduced here is that sudden discontinuities in the frequency plane introduce false oscillations (ripple) in the inverted scan and these ripples can be mistaken for lesions. They have the appearance of a smooth periodic undulation across the scan and are particularly apparent at a sharp edge of the organ or phantom (Fig. 2). Engineers have expended a great deal of effort designing filters which minimize this ripple (*9*), but none entirely satisfactory for our purpose has been developed. They involve "cutting off" the filter function gradually instead of abruptly. We have tested a roll-off characteristic based on the Dolph-Chebyshev function developed in antenna design for this purpose (*10*) but more recently have used a polynomial function which trails off more gradually (Fig. 3).

Below one cycle per inch the filter function consists of the reciprocal of the modulation transfer function of the particular collimator and nuclide in use and is approximated by a Gaussian function. The exact point in the frequency plane at which it is best to discontinue this function and begin to introduce the smoothing function has still not been decided, but good results have been obtained with

BEFORE LINEAR FILTERING WITH CUTOFF AT 1.0 CPI GENTLE ROLLOFF 3 DISKS

AFTER LINEAR FILTERING WITH CUTOFF AT 1.0 CPI GENTLE ROLLOFF 3 DISKS

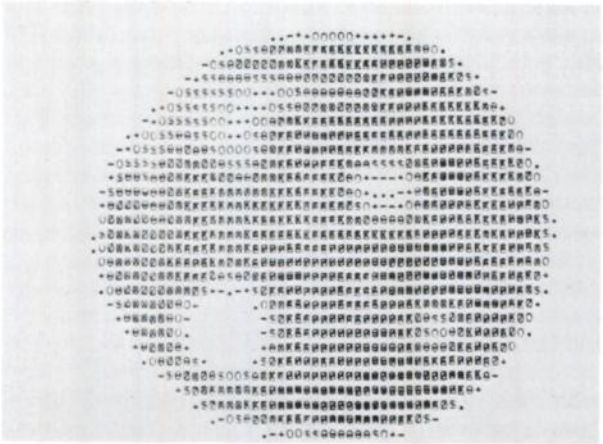

FIG. 4. Radioactive 5-in. disk overlaid with four lead disks—¾ in. at lower left, ½ in. at upper right, ⅜ in. at lower right, and ¼ in. at upper left. Latter coincided with nonuniform area of disk. Upper scan is before and lower after transform filtering.

cutoffs around 1 cycle/in. Our studies show that although the actual distribution cannot be reconstructed, we can approximate the original distribution closer than does the observed scan. Resolution enhancement in scans of both phantoms and patients has been achieved (Figs. 4 and 5), but we have not yet quantitated the degree of improvement. Studies designed to accomplish this are in progress. We are also developing and testing improved filter functions which will further reduce ripple without degrading resolution enhancement.

METHOD

Our method of recording and processing scans follows (*11–14*). Pulses from the pulse-height analyzer of a modified 5-in. crystal rectilinear scanner are accumulated in an 8-bit binary scaler. A shaft encoder is attached by gears to the beam of the scanner. As the crystal traverses the patient, at optional distances of 0.025, 0.05, or 0.1 in., a process interrupt is generated which causes the accumulated count to be transferred to a buffer and resets the scaler to zero to resume a new count. The process interrupt also causes the computer, a disk-oriented IBM 1800 system operating in a multiprogramming mode, to store in its memory the buffer count along with a 7-bit position word generated by the encoder and an internal clock reading. Ends of lines are also recorded. Scan recording, processing, and display are all controlled from a remote typewriter keyboard. After accumulating 160 scan samples in core, the data are transferred to a disk. Upon completion, the scan is automatically rearranged reversing every other line, correcting for variations in scanner speed using the clock reading, and correcting for margin shifts during the process of scanning. The scan is then stored in a file on another part of a disk to await processing. About 35 scans can be stored here. The scans are finally transferred to tape for permanent storage. The data from a scan usually occupy about a 100 $\times$ 100 element array, and for most scans each element has spatial dimensions of $\frac{1}{10} \times \frac{1}{6}$ in. (standard printer character dimensions). Two modified versions of the Fast Fourier Transform Program are in use—one disk oriented for the IBM 1800 and one "in-core" for a CDC 6400. The 1800 program uses data directly from the disk. For the 6400 program the scan data are dumped from the 1800 onto cards and transmitted through a microwave link to the 6400 computer located 40 miles away, output being returned through the link to a printer.

Fast Fourier Programs are most efficient when using arrays with dimensions in powers of two. Therefore the data array is filled out with zeros to form a 128 $\times$ 64 or 128 $\times$ 128 matrix. This method of filling in the scan edges with zeros can cause ripple in the final result if the scan values are not approximately zero at the edge because of the sharp change from non-zero to zero values. A smooth extrapolation at the edges would perhaps be preferable, but to date the insertion of zeros has been satisfactory (*15*). Next the two-dimensional transform of the completed matrix is computed, and the deconvolution filter function is applied. Since the Fourier transform of real data is complex conjugate symmetric, we need to consider only the first half of a complex array. Missing values may be obtained by complex conjugation. The transform is multiplied point by point by the filter function, achieving division by the transform of the scanner response function and smoothing and then inverse transformed. The program allows transformation of a scan 128 $\times$ 64 elements, filtering, and inversion with printer output of the processed scan in 20 sec of central processor time on the CDC 6400 computer. Eighteen minutes is required on the 1800 computer. New hardware devices for direct Fourier transformation of data have been developed recently and when these become available they should allow much faster computing times.

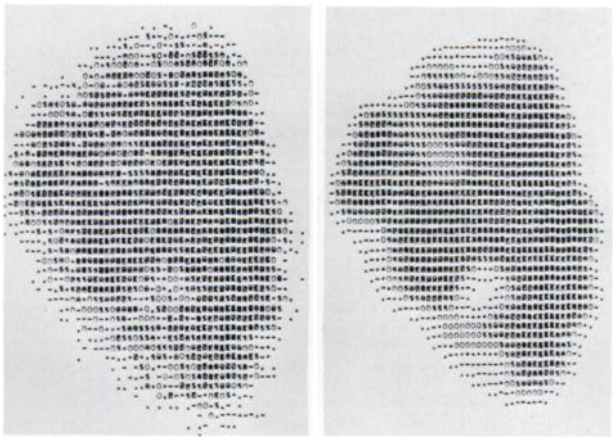

FIG. 5. Right lateral liver scan of patient with known liver metastäsis. Scan on left is before and one on right after Fourier Transform Filtering (C.D. = 800 counts/cm^2).

DISCUSSION

Many other transformations and filtering methods should be investigated. We are presently studying a nonlinear method—so-called "homomorphic filtering". It involves taking the logarithms of the data points, then taking the Fourier transform, linear filtering, inverse transforming, and taking the antilogs to obtain the processed scan (Fig. 6). A variation on the homomorphic filter which we plan to investigate involves raising the data points to an exponential power before deconvolution followed by taking the

BEFORE FILTERING DISK PHANTOM

AFTER FILTERING DISK PHANTOM

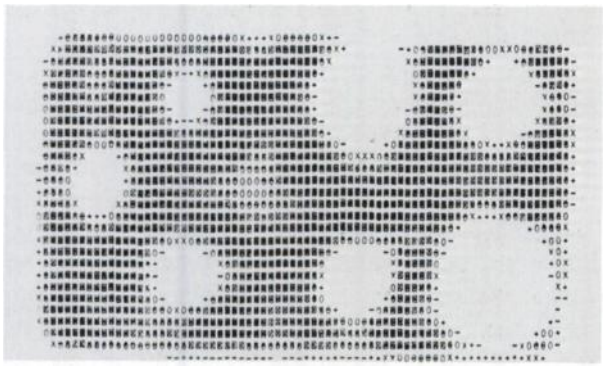

FIG. 6. Scan of Picker liver phantom made with ^{99m}Tc in air before and after homomorphic filtering (C.D. = 800 counts/cm^2).

natural logarithm of the result. It is anticipated that this variation on the homomorphic technique will partially compensate for the nonlinear response of the human eye to a uniform increase in gray scale.

The Fourier transform may be used to achieve differential filtering. This is accomplished by setting the coefficients of the lowest and highest frequencies to zero and using a linear ramp function increasing at higher frequencies to multiply the coefficients of the intermediate frequencies (*9*). We have applied differential filters to both phantom and patient scans. So far the results appear to be similar to those achieved with contour plotting of smoothed scans.

We are also studying the Hadamard transform (*16*). It differs from the Fourier in that it represents the scan as a combination of square waves rather than sinusoids. Its chief theoretical advantage is that transformation, filtering, and inversion can be carried out using integer rather than floating-point arithmetic, resulting in greatly shortened computer times. It also offers the best hope of allowing resolution enhancement of this type to be carried out on the small dedicated computers which are being installed in most nuclear medicine laboratories. We have transformed scans with the Hadamard transform (Fig. 7) and achieved resolution in phantom scans nearly as good as that achieved with the Fourier method. We have not yet written a "fast" Hadamard program with the same algorithm used in our Fast Fourier Program.

Organs normally vary greatly in size and shape. It is our belief that the computer can assist the physician in differentiating these normal changes from lesions. Others have investigated this possibility but have used standard statistical methods applied in the spatial domain (*17–19*). Our approach is to study the frequency distributions of organs. In the frequency domain the magnitudes of the coefficients (square root of the sum of the squares of the real and imaginary values) describe the important characteristics of an organ. Since we are dealing with magnitudes of complex coefficients, and these are independent of phase, the location of the organ in the data array will not matter, at least in theory. We have collected a normal group of 20 liver scans and are presently determining their transform magnitudes. We plan to reduce the normal liver pattern to a small array of low-frequency magnitudes and compare these to those in the transforms of new liver scans as a test for normalcy. Admittedly, this work is preliminary, but we believe it is very promising on a theoretical basis.

BEFORE HADAMARD FILTERING

AFTER HADAMARD FILTERING

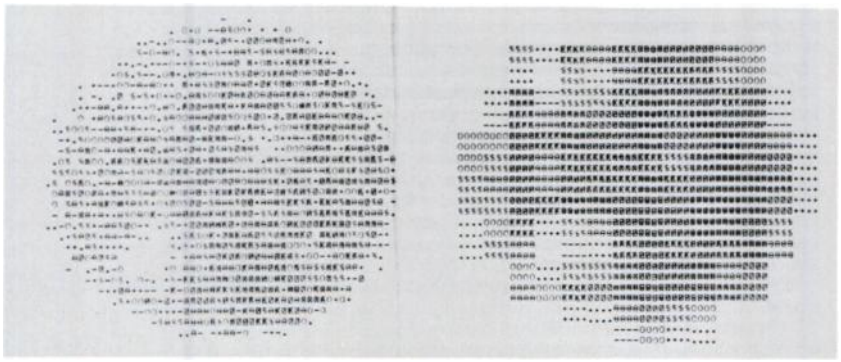

FIG. 7. Data in Fig. 4 is shown here filtered using Hadamard transform.

SUMMARY

Methods of achieving resolution enhancement of radionuclide scans by computer processing are being investigated. A digital array of scan data is transformed from the spatial to the frequency domain using a two-dimensional Fast Fourier Transform algorithm. A frequency domain filter which combines smoothing and deconvolution with the modulation transfer function of the scanning system is then applied, followed by inversion back to the spatial domain. Differential, homomorphic, and other filtering techniques, as well as Hadamard transforms, are also being studied. Resolution enhancement of phantom and patient scans has been achieved, but the degree of improvement not yet quantitated.

ACKNOWLEDGMENT

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