COMPUTER-FOCUSING USING AN

APPROPRIATE GAUSSIAN FUNCTION

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Because the conventional radioisotope image can be regarded as the convolution of true radioisotope distribution in tissue and the system-response function of the imaging system used, one can make the blur in an observed image introduced by poor resolution much smaller by solving the convolution integral equation. Efforts have been made to solve this complex problem using a digital computer (1–6).

The "computer-focusing" that we have reported (5–7) was based on the iterative approximation principle as a mathematical procedure of the image restoration. However, this "iterative approximation method" required considerable computer time. Therefore the computation cost was expensive. For this reason, it may be worthwhile to establish a less time-consuming method of deconvolution.

In this report, we describe a new method of image restoration in area scans which we call the "differential-operator method." The point-spread function of the imaging system can be described approximately by an axially symmetric Gaussian function. The convolution integral equation with Gaussian kernel function is solved using the Fourier integral representation of the Gaussian function. This is the mathematical foundation of the differential-operator method.

The mathematical procedures for the differential-operator method are as follows: first, the second-order moments of the observed point-spread function are calculated; then the backward Cauchy problem is solved using the exponential of the Laplacian operator of the moments.

That is,

$$M_{20} = \int\limits_{-\infty}^{\infty} \int R(xy)\, x^2\, dx \qquad (1)$$

$$M_{11} = \int\limits_{-\infty}^{\infty} \int R(xy)\, xy\, dxdy \qquad (2)$$

$$M_{02} = \int\limits_{-\infty}^{\infty} \int R(xy)\, y^2\, dy. \qquad (3)$$

Then the Laplacian operator of the moments is applied to the observed "digital image."

$$F(xy) = e^{-\Delta M}\, G(xy) \qquad (4)$$

in which

$$\Delta M = \frac{1}{2M_{00}}\left(M_{20}\frac{\partial^2}{\partial x^2} + 2M_{11}\frac{\partial^2}{\partial x\partial y} + M_{02}\frac{\partial^2}{\partial y^2}\right)(5)$$

and

$$e^{-\Delta M} = \left(1 - \Delta M + \frac{\Delta M^2}{2!} - \frac{\Delta M^3}{3!} + \frac{\Delta M^4}{4!}\cdots\right)(6)$$

M_{20}, M_{11} and M_{02} are the second-order moments calculated from the observed point-spread function which has nearly a Gaussian shape but is not necessary by a Gaussian function. Δ is the generalized Laplacian, and $e^{-\Delta}$ is the inverse operator—the restoration operator. $R(xy)$ and $G(xy)$ are the resolving-power array of the imaging system to a point source and the observed original image under consideration, respectively. The $F(xy)$ can therefore be considered the mathematically focused image. This means that only the blur in the observed original image due to Gaussian dispersion is compensated by this method.

To investigate the practical effectiveness of this new image-restoration method, a preliminary experiment was conducted using digital scan data from a point source (2 mm² in dia) and a paper thyroid phantom obtained using a commercial rectilinear scanner with a honeycomb collimator. An "elemental image" was selected as counts in a 1 mm² area, adjusting scanning speed of the scanner and counting time of a multiscaler. The detail of the experimental methods has been described previously (5).

The computation for the differential-operator method is performed with an IBM-7090 digital computer by using Fortran. The second-order moments were calculated from a central part of the smoothed and normalized resolving-power array that consisted of a circular array with a radius of 15 elemental images. The numerical values of the moments obtained were

$$M_{00} = 2.536673 \times 10^5$$

$$M_{20} = 8.446870 \times 10^6$$

$$M_{11} = 4.152669 \times 10^4$$

$$M_{02} = 8.570888 \times 10^6.$$

Here, M_{00} is the total sum of the elemental images in the circular array. M_{11} is much smaller than M_{20} and M_{02}, and M_{02} is nearly equal to M_{20}, suggesting that $R(xy)$ is almost symmetrical to the center of the array. Consequently, M_{11} is neglected in calculating Eq. 5. A first-approximation calculation of Eq. 6 is satisfactory. Thus the following approximations are allowed for practical purposes.

$$M_{20} \doteqdot M_{02}$$

$$M_{11} \doteqdot O.$$

Equation 4 can then be rewritten as follows:

$$F(xy) = G(xy) - \frac{1}{2M_{00}}\left(M_{20}\frac{\partial^2 G}{\partial x^2} + M_{02}\frac{\partial^2 G}{\partial y^2}\right)$$

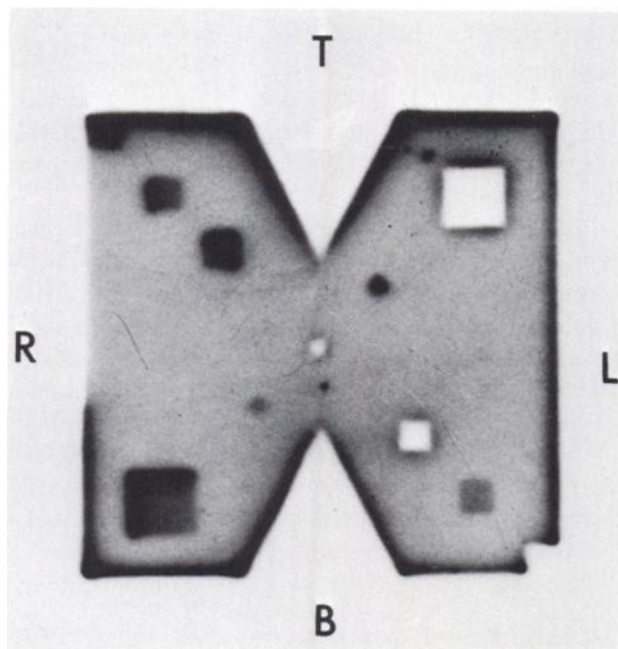$$= G(xy) - a\left(\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}\right) \qquad (7)$$

where

$$a = \frac{M_{20}}{2M_{00}} \doteqdot \frac{M_{02}}{2M_{00}}.$$

Using these simplifications, the differential-operator method is applied to the resolving-power array of the imaging system to the point source. The restored point-spread function yields the following numerical values.

$$M_{00} = 2.698475 \times 10^5$$

$$M_{20} = 4.498591 \times 10^6$$

$$M_{11} = 1.078584 \times 10^5$$

$$M_{02} = 4.857602 \times 10^6.$$

The total sum of the data, M_{00}, is maintained only slightly different from the original M_{00}.

To visualize the effectiveness of this procedure, the width at half maximum of the transverse profile curves of the resolving-power array through the center was compared before and after mathematical focusing. The first approximation resulted in sufficient sharpening of the point-spread function (approximately ½ of the observed original half-maximum width) and yielded a slightly smaller half-maximum width than did the third iteration of the iterative approximation method. Thus first approximation of the differential-operator method was satisfactory for restoring the image of the point source.

Moreover, the differential-operator method needed much shorter computer time (about 2 min) than did the iterative approximation method (about 30 min).

As the next step, the digital scan data of the thyroid phantom was used for mathematical focusing. Fig. 1 shows an autoradiogram of the phantom. Fig. 2 gives a smoothed image of the phantom which was derived from the observed original image by replacing the central elemental image with an average value of eight neighboring elemental images.

The focused image calculated with the differential-operator method is shown in Fig. 3. It can be compared with the same pattern for the focused image made with the iterative approximation method in Fig. 4. Both focusing calculations were followed by the smoothing calculation. These images were plotted as isocount contour curves at 5% intervals from 30% to 100% using a Calcomp-570 x-y plotter and then pictured manually in different color bands. The relationship of color and percentage value is also shown in the figures.

It can be concluded that the blurred image of the phantom is restored effectively by both methods of mathematical focusing. The image obtained with the differential-operator method seems to be focused more sharply than that with the iterative approxi-
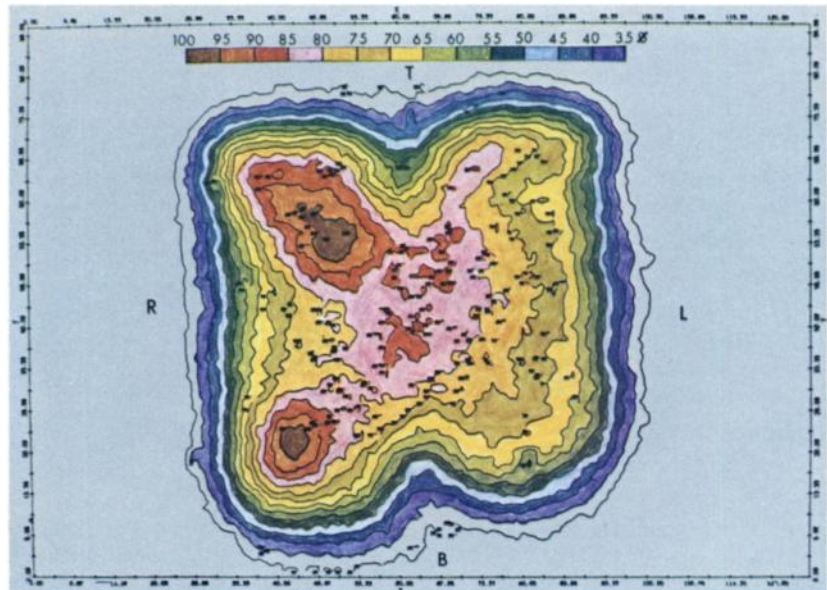
FIG. 2. Isocontour display of smoothed image. Isocontour lines are drawn at 5% intervals between 30% and 100% using Calcomp x-y plotter. Display is pictured manually in different color bands to visualize effect more clearly.
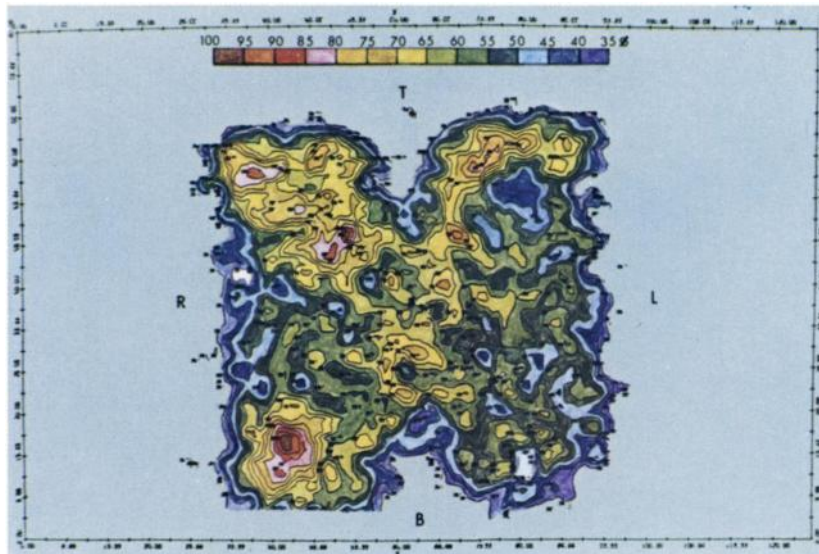

FIG. 3. Isocontour display of focused image calculated with differential-operator method.
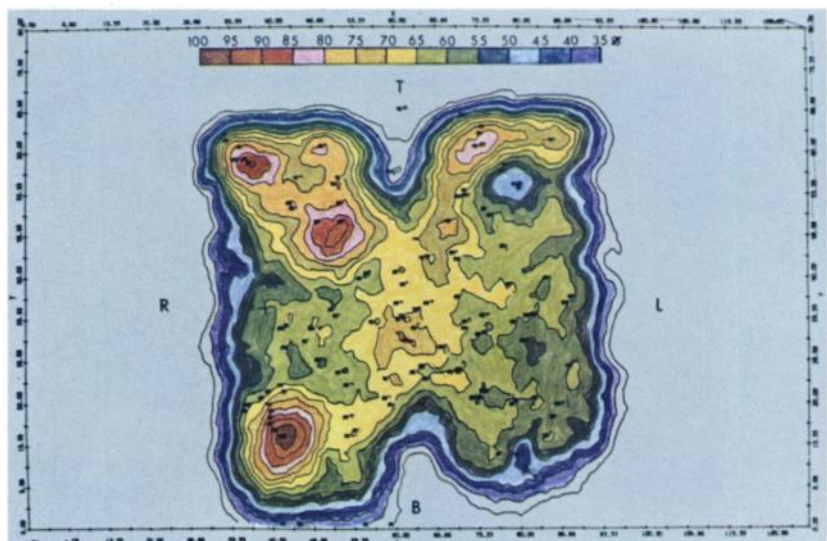

FIG. 4. Isocontour display of focused image calculated with iterative approximation method.

mation method. The former, however, contains more mottling than does the latter, suggesting that the former is more sensitive to statistical fluctuations included in the original data. To minimize the artifact associated with the oscillatory behavior, we had to smooth again before plotting.

Although further investigations are necessary to determine which of these two mathematical focusing methods is better, one can conclude that the differential-operator method is less difficult to program for computer processing and is less time consuming. Moreover, it may be applicable to on-line real-time data processing, while the iterative approximation method is less effective for deconvolution and is not suitable for the on-line system. One might be able to build an analog computing circuit based on a similar mathematical principle in radioisotope imaging devices themselves.

The detailed mathematical background of this differential-operator method will be reported elsewhere (7,8).

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