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EDITORIAL

Scattered Photons as "Good Counts Gone Bad:" Are They Reformable or Should They Be Permanently Removed from Society?

In general, the quality of an image can be described (quantitatively) by its signal-to-noise ratio (*1*), which directly affects diagnostic and quantitative accuracy. The signal-to-noise ratio describes the relative "strength" of the desired information and the noise (due to the statistics of radioactive decay, for example) in the image. The signal is typically thought of as the difference or contrast between a target and the surrounding activity. In practice, this contrast is provided in the patient by the radiotracer's distribution. The goal of the imaging system is to preserve this contrast in the image. Contrast is maintained by avoiding blurring, which smears counts from higher-activity regions into lower-activity regions (and vice versa), thus reducing image contrast. Therefore, spatial resolution, in its broadest sense, and contrast are closely linked. This relationship is quantitatively described by the imaging system's modulation transfer function, which is the Fourier transform of the point spread function. While the modulation transfer function is obtained from a conventional measure of spatial resolution, it is actually the ratio of the contrast in the image to that in the object as a function of spatial frequency (*2*). Inclusion of scattered photons in the image reduces contrast; this is partially reflected in a change in the point spread function and modulation transfer function (*2*). The amount of scatter depends on the distribution of activity within the patient, the patient's body habitus, the imaging geometry of the system, the system's energy resolution and the pulse height window setting.

The design of a PET or SPECT system must address these issues by attempting to simultaneously maximize spatial resolution and sensitivity, while minimizing the acceptance of scattered photons. In practice, these competing design goals lead to an "optimum" (in the designer's mind) compromise, and real-world scanners have less-than-ideal resolution, sensitivity, and scatter characteristics. There is, thus, much interest in software-based postacquisition approaches to these problems. For the sake of simplicity, many software approaches begin with the assumption of a linear, shift-invariant system. Such a system responds linearly to changes in activity distribution regardless of the position of the activity within the field of view. In such a situation, the measured projection data can be considered as the convolution of the object with the imaging system's response:

$$p = o * h, \qquad \text{Eq. 1}$$

where p represents the projection data, o the object and h the imaging system's response (i.e., the point spread function). The asterisk represents convolution. It is important to note that h contains both resolution and scatter effects. The convolution theorem states that convolution in real space is equivalent to multiplication in Fourier space. If we use capital letters to denote the Fourier transform of a function, the above equation thus becomes:

$$P = O\,H. \qquad \text{Eq. 2}$$

In such a situation, o can be obtained from p by deconvolution with a known h (i.e., based on a measurement of a point source). Deconvolution is usually performed in Fourier space, where mathematically it is a simple division:

$$O = P/H, \qquad \text{Eq. 3}$$

in which o is obtained from O by taking the inverse Fourier transform. H^{-1} is known as the inverse filter. In the absence of noise, such a filter will perfectly restore a blurred projection. In practice, the use of such a filter would lead to unacceptably large noise amplification, and a combination of inverse filtering and low-pass filtering must be used. This approach forms the basis for all Fourier-based restoration filtering (e.g., Wiener or Metz filtering) in nuclear medicine. Such filters usually are composed of an inverse component (i.e., a boost) at low to intermediate spatial frequencies, followed by a roll-off (i.e., a cut) at intermediate to high spatial frequencies. Since scatter is mainly though by no means exclusively a low spatial frequency phenomenon, I have previously argued that the main effect of such filtering is scatter reduction, by the equivalent of deconvolution. Of importance, deconvolution here reduces scatter through a process of repositioning of scattered events, not by their elimination (*3,4*).

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Other investigators have chosen to subtract an estimate of the scatter distribution from the observed data. In SPECT, this approach was pioneered by Jaszczak et al., who used a second pulse-height window positioned over the Compton region of the pulse-height spectrum to estimate the scatter distribution (*5*). More sophisticated approaches with the same theme have now been proposed (*6*). In PET, a similar approach to estimation followed by subtraction was pioneered by Bergstrom and colleagues (*7*). In their approach, the measured projection data are treated as true, scatter and random coincidence events:

$$p = t + s + r. \qquad \text{Eq. 4}$$

After correction for random coincidence events, the measured projection data consist of true plus scattered events. Bergstrom modeled scatter as the convolution of the true radiotracer distribution with a scatter distribution function h_s:

$$s = t * h_s. \qquad \text{Eq. 5}$$

In such a situation, the measured projection data become:

$$p = t + (t * h_s). \qquad \text{Eq. 6}$$

In order to directly obtain the true distribution t, the above equation could be rearranged:

$$t = p - (t * h_s). \qquad \text{Eq. 7}$$

This approach requires a priori knowledge of t, the true distribution which we ultimately wish to know. Bergstrom hypothesized (and subsequently demonstrated) that a second scatter distribution function h_s' could be derived, such that:

$$t = p - (p * h_s'). \qquad \text{Eq. 8}$$

This approach forms the basis for the article by Bentourkia et al. in this issue of the *Journal* (*8*). Of importance, Bentourkia and colleagues have chosen to explicitly separate object scatter, collimator scatter and detector scatter. Of even greater significance, they have implemented their approach as a nonstationary (often called shift-variant) convolution. By so doing, they are not forced to make the assumption, particularly incorrect for ring geometry PET tomographs, that h_s is the same everywhere within the field of view.

I am particularly intrigued by the exact formulation chosen by Bentourkia et al. Like Bergstorm, Bentourkia et al. begin by defining the measured projection data after random correction, as the sum of individual scatter components:

$$p = t + s_o + s_c + s_d. \qquad \text{Eq. 9}$$

They then associate a separate (shift-variant) h with each component. I might have expanded Bergstrom's equation to separately consider object, collimator, and detector scatter as:

$$t = p - (p * h_o) - (p * h_c) - (p * h_d) \qquad \text{Eq. 10}$$

$$= p * (\delta - h_o - h_c - h_d),$$

where δ is the Dirac function as formally defined. Bentourkia et al. have used a different formulation in which the projection data, as modified by the current scatter correction step, are used as the input for the subsequent step. That is, the object-scatter corrected data are used in the collimator-scatter correction step and the object- and collimator-scatter corrected projection data are used in the detector-scatter correction step. As stated in the article by Bentourkia et al., the final equation becomes:

$$t = p * (\delta - h_o) * (\delta - h_c) * (\delta - h_d). \qquad \text{Eq. 11}$$

A creative aspect of this approach is that each successive subtraction stage builds on the previous stages, and, in that sense, the operations are not independent.

Bentourkia et al. were able to demonstrate improvement in image contrast with their approach, and the importance of separating object, collimator and detector-scatter components in the correction scheme. They were further able to show that subtracting detector scatter was undesirable because it lowered signal (or increased noise, depending on your point of view). They were not able to demonstrate significant resolution recovery. I am not surprised by this finding, since the object and collimator scatter distribution functions are essentially low-spatial frequency in character. We have equivalently demonstrated that Fourier filtering-based scatter reduction mainly affects the tails of the point spread function (*3,4*).

In their concluding remarks, Bentourkia et al. highlight the difficulties in improving image signal-to-noise ratio. While their method improves contrast, and thus signal overall, it does so at the expense of noise, because it involves the removal by subtraction of counts. This is in marked contrast to deconvolution methods like Fourier filtering, which can simultaneously reduce scatter and noise, because no counts are removed (*3,4*). Like Bentourkia et al., I believe that a restoration model capable of preserving (recovering) the geometric resolution, removing scatter and suppressing noise is required in high-resolution PET. The approaches to resolution recovery, scatter removal and noise suppression within such a model will likely influence each other. With respect to scatter correction, a continuing challenge to the developers of such a model will be the choice to reposition or remove scattered counts from the image. Such a choice will clearly influence more than just image contrast.

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