APPENDIX

As corrected elements X^i approach the true distribution, the value of the convolution integral $\Sigma A_{kl}\, X^i_{ij}$ also converges to the value of the original element X_{ij}. However, one should not attempt to approach the true distribution to a greater degree of precision than that with which the original elements X are measured. This will only lead to the generation of false oscillations in the solution which was, indeed, the main difficulty encountered in applying the Fourier-transform method to the restoration of degraded optical images (*23*). It was shown mathematically that the noise inherently present in the images could result in such serious oscillations in the restored images that the solution was physically meaningless. The same phenomenon can occur in the radioisotope images because they are noisier than the optical images.

Consequently, in our study the approximation was stopped if $\Sigma A_{kl}\, X^i_{ij}$ agreed with X_{ij} within one statistical standard deviation of X_{ij}; that is $\pm\ (X_{ij})^{1/2}$. Then X^i should be taken as an adequate approximation to the true distribution.

The above equality can be written:

$$|X_{ij} - \Sigma A_{kl} X^i_{ij}| \equiv \epsilon_{ij} \leqq (X_{ij})^{1/2} \qquad \text{(A-1)}$$

$$\frac{(\epsilon_{ij})^2}{X_{ij}} \leqq 1\ (i = 1, \ldots n, j = 1 \ldots m) \qquad \text{(A-2)}$$

If we sum this over all elements, we obtain Eq. A-3

$$\sum_{ij} \frac{(\epsilon_{ij})^2}{X_{ij}} \leqq nm \qquad \text{(A-3)}$$

In Eq. A-3 it should be noted that X_{ij} agrees with $\Sigma A_{kl}\ X^i_{ij}$ within the limit of $\pm\ (X_{ij})^{1/2}$ on the average although criterion of the limit (Eq. A-1) may not be satisfied at every element.

If the error ϵ_{ij} has an identical value over all elements, the criterion defined by Eq. A-1 is satisfied when Eq. A-3 holds. However, the ϵ_{ij}'s are generally different between the elements, and the speed of convergence seems to depend on the distribution of the original elements X_{ij}. Actually it is possible to continue the approximation until the criterion of Eq. A-1 is satisfied with all the elements, but the solution tends to contain false oscillations in these cases. Therefore the limit defined by Eq. A-3 (that is Eq. 3) is regarded as a satisfactory compromise between the true distribution and the false oscillations in the solution, and thus the over-all accuracy of the focused elements is also determined by the same equation.

It is difficult to verify analytically the convergence of Eq. A-3 which can usually be observed only experimentally. However, it should be mentioned that smoothness of the original image (experimentally observed image) is one of the important factors to assure fast convergence of Eq. A-3 and thus to obtain a satisfactorily focused image. Consequently, the original elements must be made as smooth as possible before iterative approximation is initiated.